# Introduction of JMA-NHM

Hiroshi Kusabiraki (JMA/NPD)

Training Workshop on the Latest Developments on the Use and Interpretation of NWP Models

HKO, 3-7 Dec 2012

# Contents

Introduction of JMA-NHM

# 0. WHAT IS A NUMERICAL MODEL ?

# What is a Numerical Model?

- Numerical weather prediction uses mathematical models of the atmosphere and oceans to predict the weather based on current weather conditions.

(by wikipedia ☺)

mathematical equations that describe the "dynamics" and "physics" of the atmosphere

to solve those equation, numerically

⇩

get the value of atmospheric condition in the future

# Numerical Model

Equation describing atmosphere

$$\frac{\partial \phi}{\partial t} = F$$

Time tendency

Differential form

$$\phi_{\mathrm{t}+\Delta \mathrm{t}} = \phi_{\mathrm{t}} + F_{\mathrm{t}} \Delta t$$

Future — Current

## In numerical prediction model

$$\phi_{\mathrm{t}}$$

Current state (already known)

$$F_{\mathrm{t}} = D_{\mathrm{t}} + P_{\mathrm{t}}$$

Time tendency
D: dynamics
P: physics

$$\phi_{\mathrm{t}+\Delta \mathrm{t}} = \phi_{\mathrm{t}} + (D_{\mathrm{t}} + P_{\mathrm{t}}) \Delta t$$

Time integration
(Depends on schemes)

Iterate

1. calculate time tendency
2. time integration

from initial time

→ Numerical Prediction

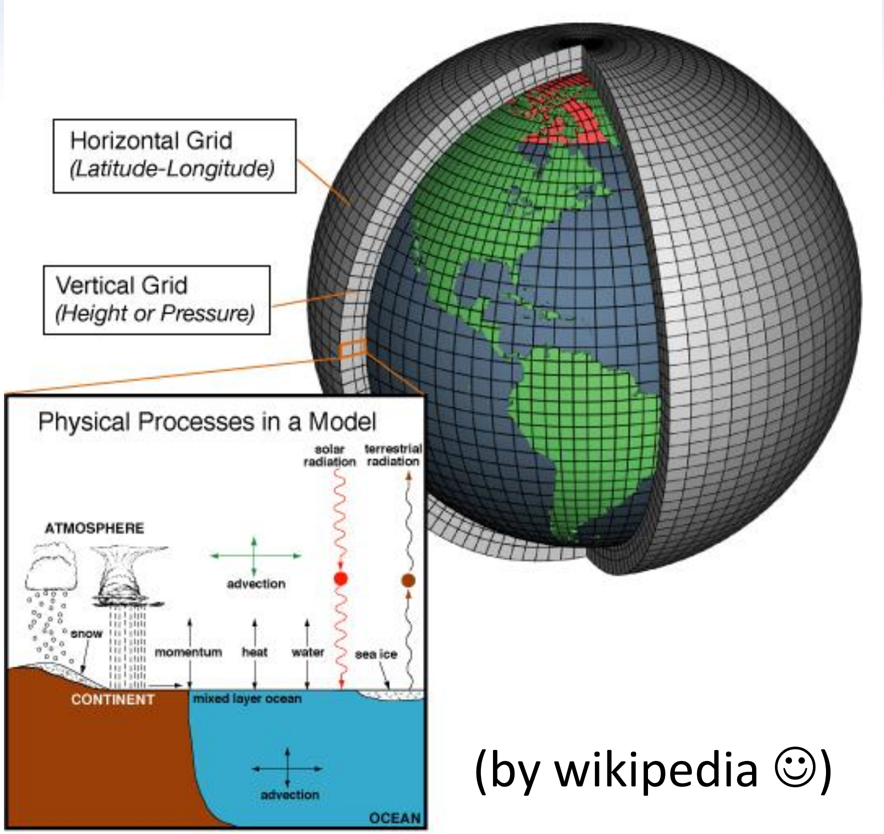
# Overview of a Numerical Model

- Dividing the Earth's atmosphere into discrete grids 3-dimensionally.

  (see right figure)
- The model calculates how much each grid affects its neighbors, and how much the atmosphere will change in each grid with time.

⇩

making a forecast




(by wikipedia ☺)

# Kind of Numerical Weather Models

- For the restricted computing power, we use the models properly.
(Even if we have the fastest computer in the world, we can't get sufficient computing power for a weather prediction.☺)
- Global Model ($\Delta$x=200~20km)
  - This model can simulate global circulation and synoptic scale phenomena.
- Regional Model ($\Delta$x=20km~)
  - Hydrostatic model($\Delta$x=~10km)
  - Non-hydrostatic model ($\Delta$x=5km~?00m)
(Cloud Resolving Model)
  - These model can simulate severe weather, but boundary conditions for them are provided by global model or another regional model.
- LES model (100~?m)
  - This model can simulate micro scale. Because it needs enormous computing power in realistic simulation, it is not a model for weather prediction.

# Limited Area Forecast Models

- There are many such models in the world.
  - JMA-NHM: The JMA's model for targeted mesoscale phenomena. (more details I will introduce later)
  - WRF: The Next-generation meso-scale numerical weather prediction system is developed by NCAR, NCEP and many researchers.
  - And, UM(UKMO), LM(=COSMO, DWD), AROME(Met. Fr.), GRAPES-Meso(CMA) …etc
  - RAMS: The RAMS was developed at Univ. Colorado. Many users switch to WRF now. (I was a user of this model during college…)

Introduction of JMA-NHM

# 1. HISTORY OF THE JMA-NHM

# Brief history (1)

- In Meteorological Research Institute (MRI/JMA) Nonhydrostatic Model (MRI-NHM) was developed by Ikawa (1984).
- On the other hand, another model was developed by Goda and Kurihara (1991) in Numerical Prediction Department (NPD/JMA) .
- A joint work between MRI and NPD was started in February 1999. A unified model (MRI/NPD-NHM) was completed in 2000.

# Brief history (2)

- Development of community model for numerical weather prediction and research was launched 2001.
- Operational run was started on 1 September 2004. Model specifications were 10km in horizontal resolution, 18hr forecast and 4 times/day. Forecast domain is 3500km x 2500km (Japan area).
- Operational run was update to 5km in horizontal resolution from 10km on 1 March 2006. It was 15hr forecast and 8 times/day with same domain.
- March 2007, we have run 33hr forecast 4 times/day and 15hr forecast 4 time/day.

Introduction of JMA-NHM

# 2. DYNAMICAL FRAME

# Dynamical Frame of JMA-NHM

- **Fully compressive equations**
  - Treatment of sound wave
  - Divergence filter
- **Advection term**
  - Modified scheme and splitting
- **Numerical diffusion**

# Governing Equations of JMA-NHM

dynamics

physics

## Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \underbrace{\nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc}}_{\text{advection}} + \underbrace{(\nabla p')^i}_{\text{pressure gradient}} + \underbrace{\left(\sigma \frac{g p'}{C_m^2} - \mathrm{buoy}\right)\delta_3^i}_{\text{buoyancy}} + \underbrace{2\rho\epsilon^{ijk}\Omega_j u_k}_{\text{Colioris}} = \underbrace{\rho \mathrm{Dif}.u^i}_{\text{diffusion by turb. scheme}}$$

advection · pressure gradient · buoyancy · Colioris · diffusion by turb. scheme

## Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ \underbrace{-\nabla_i \cdot (\rho u^i)}_{\text{divergence}} + \underbrace{\mathrm{prc}}_{\text{increase in density due to rainfall}} + \underbrace{\frac{\rho}{\theta_m}\frac{\partial \theta_m}{\partial t}}_{\text{expansion}} \right\}$$

divergence · increase in density due to rainfall · expansion

## Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho}\left\{ \underbrace{\nabla_i(\rho u^i \theta) - \theta \nabla_i(\rho u^i)}_{\text{advection}} \right\} = \underbrace{\frac{Q}{C_p \pi}}_{\text{diabatic term}} + \underbrace{\mathrm{Dif}.\theta}_{\text{diffusion by turb. scheme}}$$

advection · diabatic term · diffusion by turb. scheme

## Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho}\left\{ \underbrace{\nabla_i(\rho u^i q_n) - q_n \nabla_i(\rho u^i)}_{\text{advection}} \right\} = \underbrace{Q_n}_{\text{diabatic term}} + \underbrace{\mathrm{Dif}.q_n}_{\text{diffusion by turb. scheme}}$$

advection · diabatic term · diffusion by turb. scheme

## State Equation

$$\rho = \frac{p_0}{R\theta_m}\left(\frac{p}{p_0}\right)^{C_v/C_p}$$

# Treatment of Sound Wave

- Fully compressible equations include sound wave as a solution

Simplified equations
(2D linearized fully compressible equations)

$$\frac{\partial u^*}{\partial t} = -\frac{\partial p'}{\partial x}$$

$$\frac{\partial \rho'}{\partial t} = -\frac{\partial u^*}{\partial x}$$

$$\frac{\partial \theta^*}{\partial t} + w^* \frac{N^2}{g} = 0$$

$$\rho' = -\theta^* + \frac{p'}{c_{\mathrm{s}}^2}$$

$$(u^* = \overline{\rho} u')$$

Wave equation

$$\frac{\partial^2 p'}{\partial t^2} = c_{\mathrm{s}}^2 \frac{\partial^2 p'}{\partial x^2}$$

Speed of sound

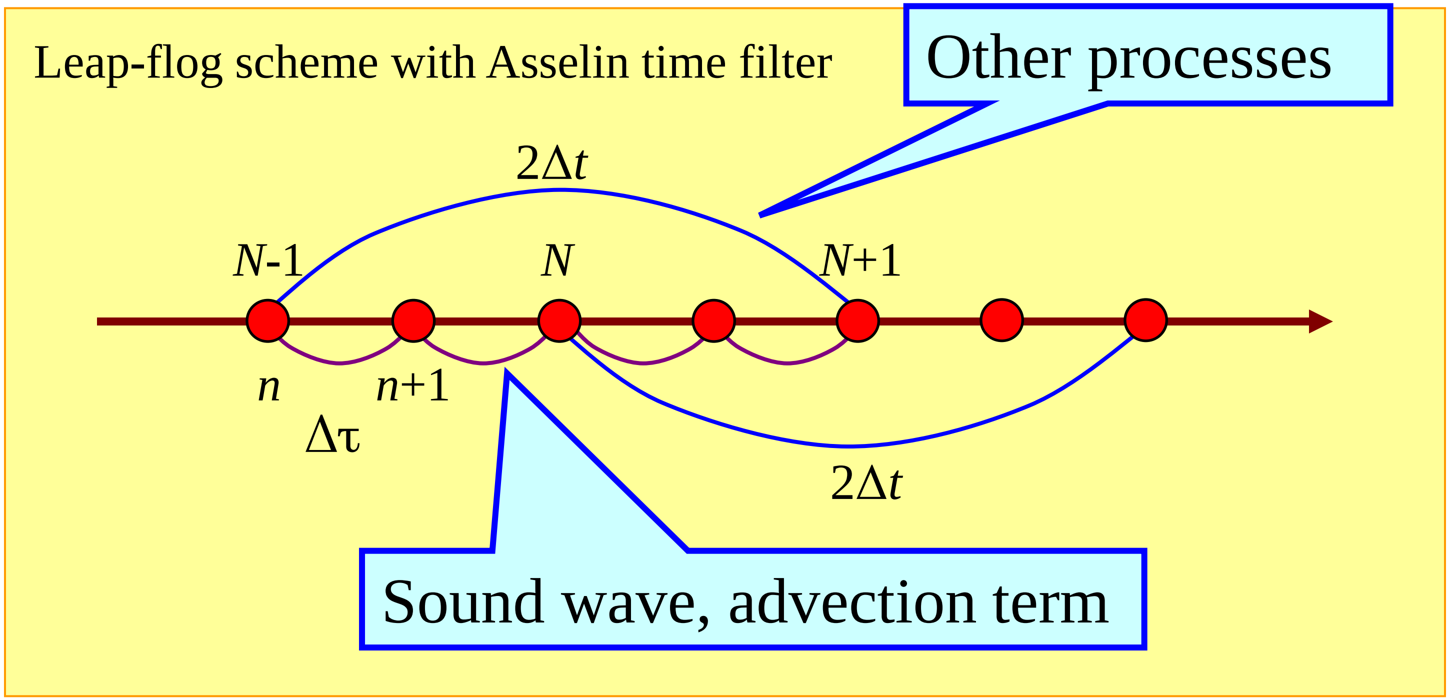
$$c_{\mathrm{s}} = \sqrt{\frac{C_{\mathrm{p}}}{C_{\mathrm{v}}} R\overline{T}} \sim 300 \mathrm{~m/s}$$

***Strong constraint against CFL condition***

$$\frac{\Delta x}{\Delta t} > C$$

# Time Integration Scheme
# - Split-Explicit Scheme or HE-VI scheme -

Explicit, but split in horizontal direction
Implicit in vertical direction



# Divergence Damping

- A case not enough computational stability by only using HE-VI
- When large Courant number for wind speed and sound wave, sound wave mode can become unstable
- Add gradient term of divergence on Z* coordinate to the momentum equations

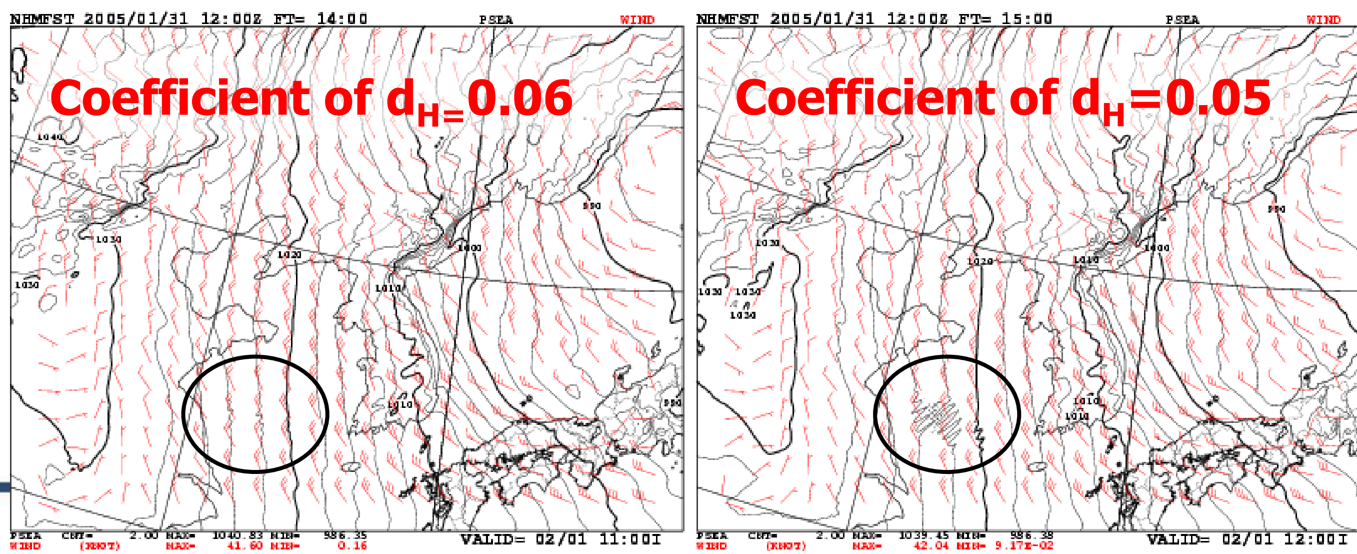
$$\frac{U^{\tau+\Delta\tau}-U^{\tau}}{\Delta\tau}+\frac{m_1}{m_2}\left(\frac{\partial P^{\tau}}{\partial \hat{x}}+\frac{\partial G^{13}P^{\tau}}{\partial \hat{z}}\right)=-\mathrm{ADVU}^{t}+\mathrm{RU}^{t}+\boxed{\alpha_{\mathrm{H}}\frac{m_1}{m_2}\left(\frac{\partial \mathrm{DIVT}^{t}}{\partial \hat{x}}+\frac{\partial G^{13}\mathrm{DIVT}^{t}}{\partial \hat{z}}\right)}$$

$$\alpha_{\mathrm{H}}=\frac{d_{\mathrm{H}}}{\Delta t}\min\left[\left(\frac{\Delta x}{m_1}\right)^2,\left(\frac{\Delta y}{m_2}\right)^2\right]$$

A case that the oscillation occurred

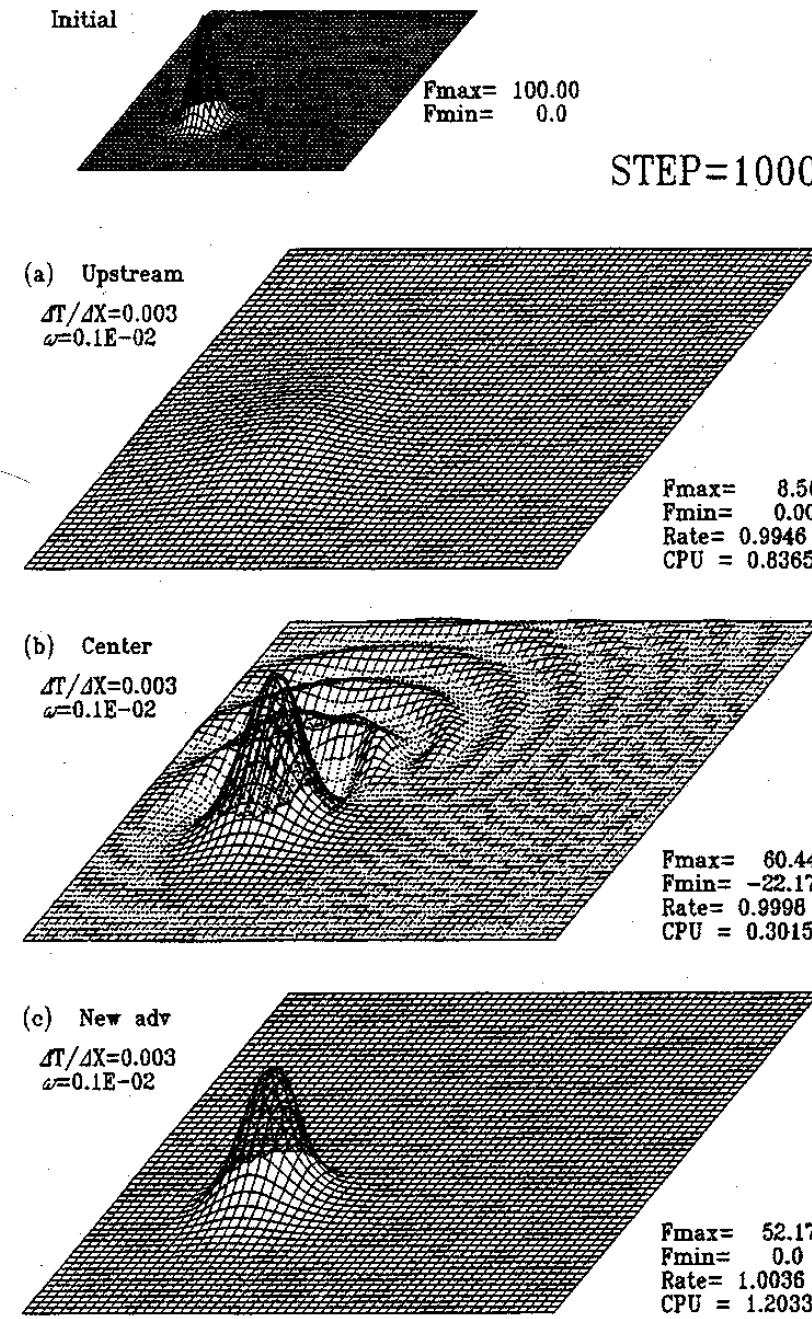
Ini. 12UTC. Jan. 31, 2005

Suppressed noise to strengthen filter



# Modified Advection Scheme

- Purpose
  - Removal of oscillation due to the computational error
- Scheme
  - 1st order difference scheme
  - After calculation of advection term, to correct it not exceeding an upper and lower limit
- Problems
  - Instead of not occurring oscillation, decreasing accuracy and decaying amplitude.

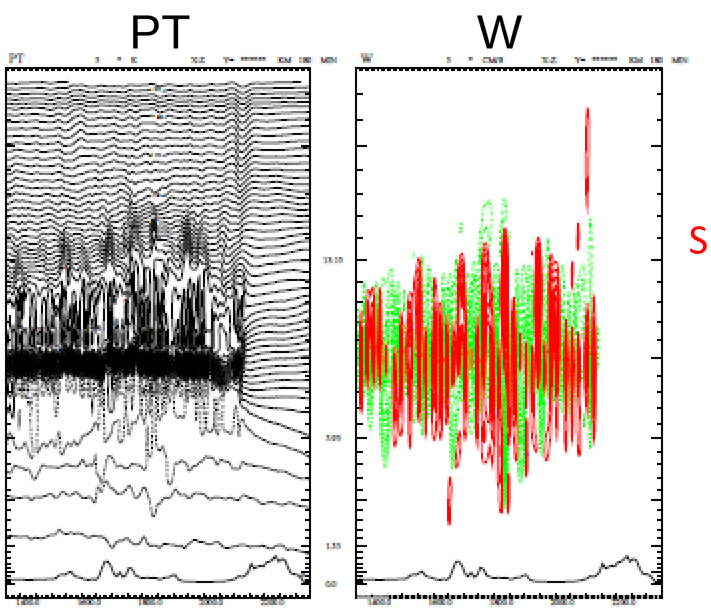



気象庁 Japan Meteorological Agency

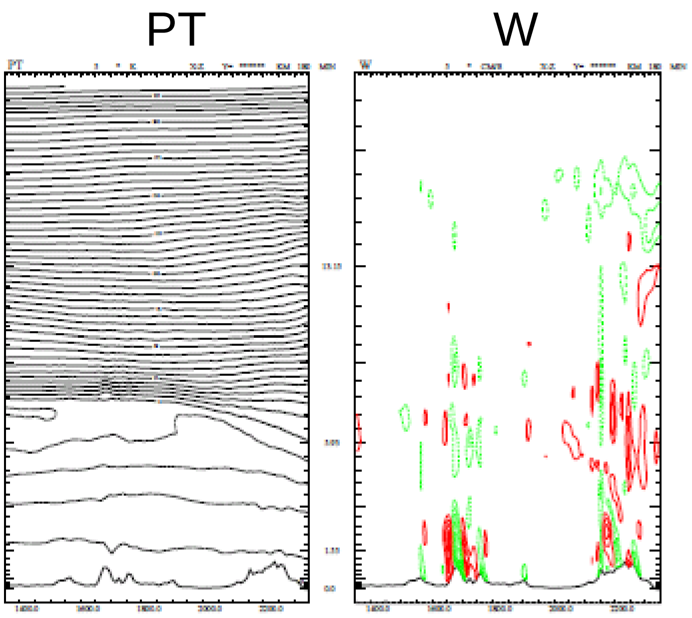
# Still Unstable Case

- Computational instability still occur, such as
  - Strong wind and stable layer
  - Deep convection

An example (Strong wind and stable layer)




Split advection term



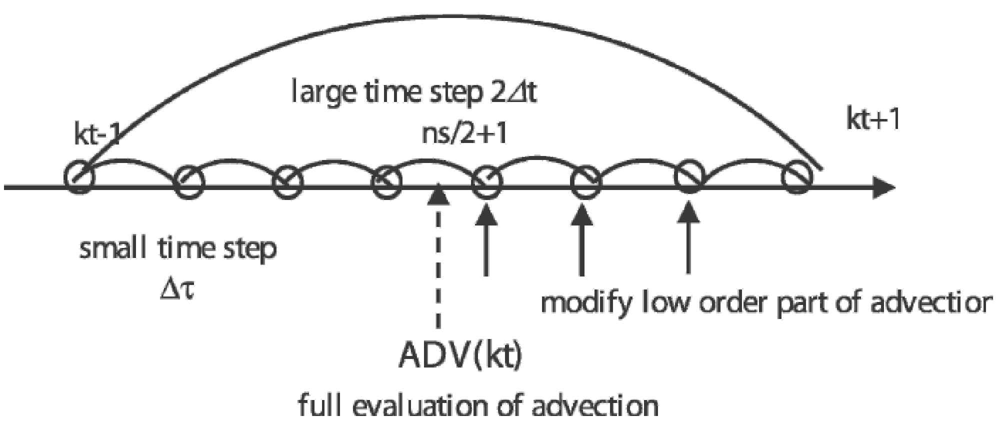

Initial : 06UTC, Apr. 9, 2002
T+3

Initial : 06UTC, Apr. 9, 2002
T+3

# Split Advection Term

- To suppress computational instability due to the jet or strong convection
- Advection term is integrated using dτ (< dt)
  - However, whole advection terms are not split. The contribution from 2nd order flux form is evaluated using dτ, in addition, advection term is corrected in the following equation
  - This correction is only adopted in the latter half of the leap-frog scheme
  - Advection scheme of PT is same as above, stabilizing gravity wave

$$\mathrm{ADV}^{\tau+\Delta\tau} = \mathrm{ADV}^{t} - \mathrm{ADVL}^{t} + \mathrm{ADVL}^{\tau}$$



# Other Adoptions for Stable Computation

- Numerical diffusion
  - Linear
    - Removal to oscillation in minimum wave length that can be resolved
    - 4th order
  - Nonlinear
- Targeted Moisture Diffusion
  - Diffuse water vapor in grids which exceeds a threshold of updraft speed.

Introduction of JMA-NHM

# PHYSICAL PROCESSES

# Physical processes

- **Transport that cannot be resolved by grid-mean velocities (i.e. subgrid transport)**
  – Convective transport
  – Boundary layer turbulent transport
- **Flow-independent flux (i.e. not transported by wind)**
  – Radiation
  – Surface flux
- **Local source or sink**
  – Latent heat release by condensation
  – Transition between hydrometeors in cloud microphysics

# Governing Equations of JMA-NHM

dynamics

physics

## Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \underbrace{\nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc}}_{\text{advection}} + \underbrace{(\nabla p')^i}_{\text{pressure gradient}} + \underbrace{\left(\sigma \frac{gp'}{C_m^2} - \mathrm{buoy}\right)\delta_3^i}_{\text{buoyancy}} + \underbrace{2\rho\epsilon^{ijk}\Omega_j u_k}_{\text{Colioris}} = \underbrace{\rho \mathrm{Dif}.u^i}_{\text{diffusion by turb. scheme}}$$

## Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ \underbrace{-\nabla_i \cdot (\rho u^i)}_{\text{divergence}} + \underbrace{\mathrm{prc}}_{\text{increase in density due to rainfall}} + \underbrace{\frac{\rho}{\theta_m}\frac{\partial \theta_m}{\partial t}}_{\text{expansion}} \right\}$$

## Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho}\left\{ \underbrace{\nabla_i(\rho u^i \theta) - \theta \nabla_i(\rho u^i)}_{\text{advection}} \right\} = \underbrace{\frac{Q}{C_p \pi}}_{\text{diabatic term}} + \underbrace{\mathrm{Dif}.\theta}_{\text{diffusion by turb. scheme}}$$

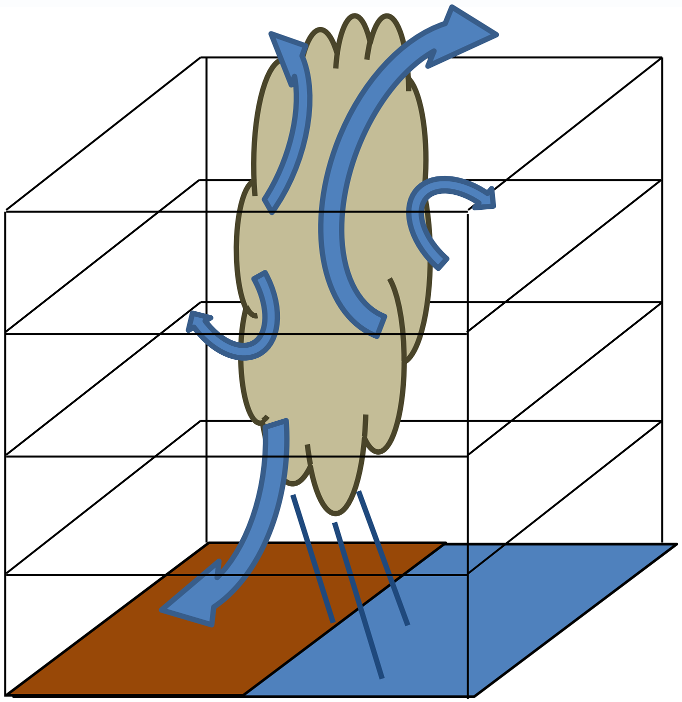
## Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho}\left\{ \underbrace{\nabla_i(\rho u^i q_n) - q_n \nabla_i(\rho u^i)}_{\text{advection}} \right\} = \underbrace{Q_n}_{\text{diabatic term}} + \underbrace{\mathrm{Dif}.q_n}_{\text{diffusion by turb. scheme}}$$

## State Equation

$$\rho = \frac{p_0}{R\theta_m}\left(\frac{p}{p_0}\right)^{C_v/C_p}$$

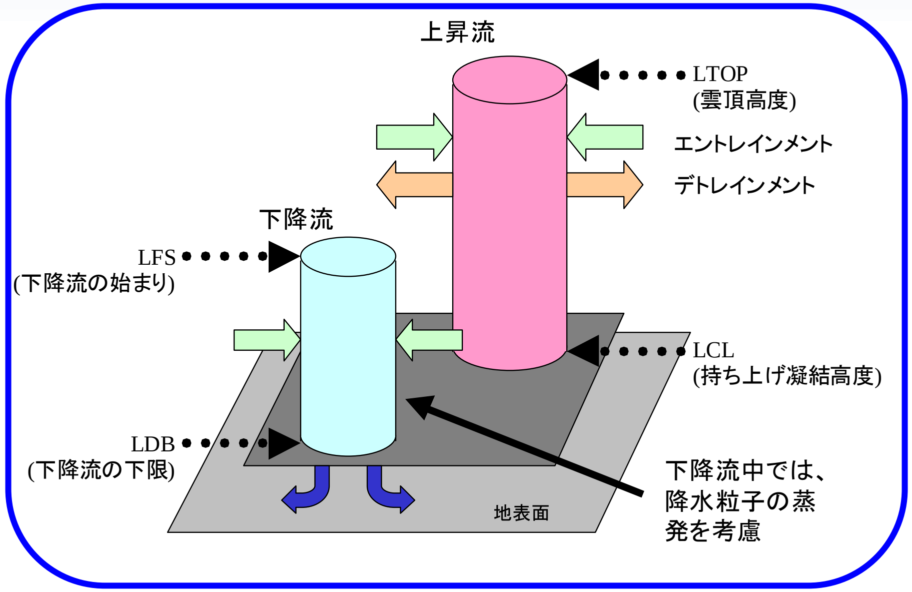
# Convective Parameterization (Subgrid Transport)



Sub-grid scale convection
- vertical mass flux
- precipitation

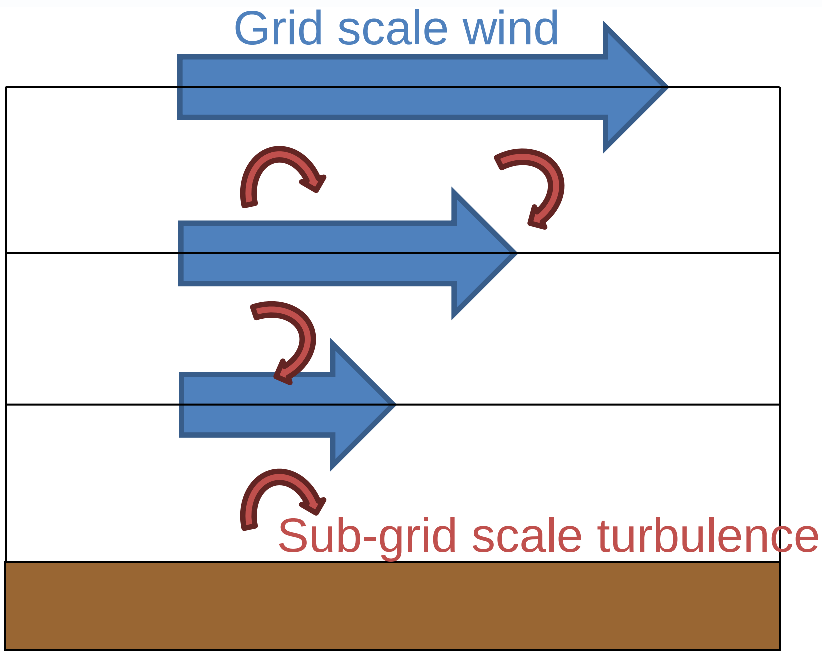
# Kain-Fritsch Scheme

- For mesoscale convective systems in midlattitude
- Trigger function
  - Convections occur at which grid, which level?
- Formulation using mass flux
- Closure assumption
  - Consume CAPE
  - Adjust mass flux




Cloud model consisted a pair of upward and downward flow, calculate
- Tendency of pt and water substances, condensation, and prep.

# Turbulence (Subgrid Transport)




Fluxes by sub-grid scale turbulence

- momentum
- heat
- water vapor

# Turbulence Scheme in JMA-NHM

- **Improved Mellor-Yamada Level 3 scheme (operational)**
  - MYNN3; Nakanishi and Niino (2006)
  - 2nd order closure model
  - Prognostic variables

$$q^2,\ \overline{\theta'^2},\ \overline{\theta' q_v'},\ \overline{q_v'^2}\ (q^2 : \text{turbulent kinetic energy},\ q_v : \text{water vapor})$$

  - Diagnostic variables

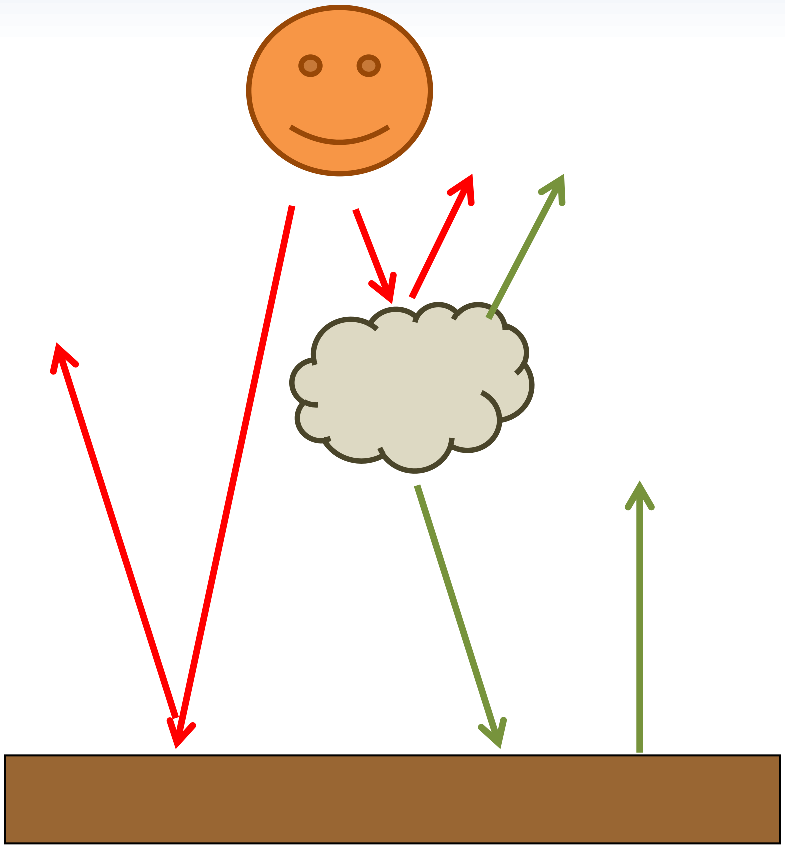
$$\overline{u'w'},\ \overline{v'w'},\ \overline{w'\theta'},\ \overline{w'q_v'},\ \overline{u'^2},\ \overline{v'^2},\ \overline{w'^2},\ \overline{u'v'},\ \overline{u'\theta'},\ \overline{v'\theta'},\ \overline{u'q_v'},\ \overline{v'q_v'}$$

JMA's original implementation

→ Expressed by diffusive form

  - Others : zero
- **Deardorff scheme (option)**
  - Deardorff (1977)

# Radiation (Flow Independent)



- Short wave radiation
- Long wave radiation

- Consider the existance of cloud
  - diagnose partial condensation scheme

- enhance the diurnal cycle of surface temperature
- Impact on the vertical profile of atmospheric temperature

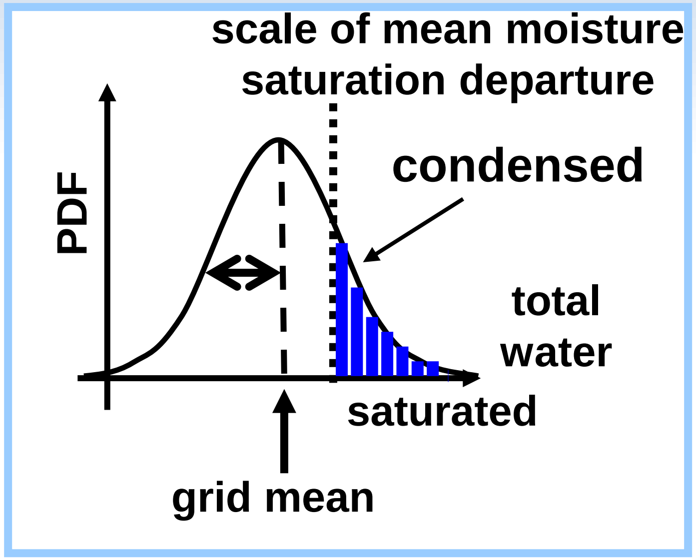
# Radiation in JMA-NHM

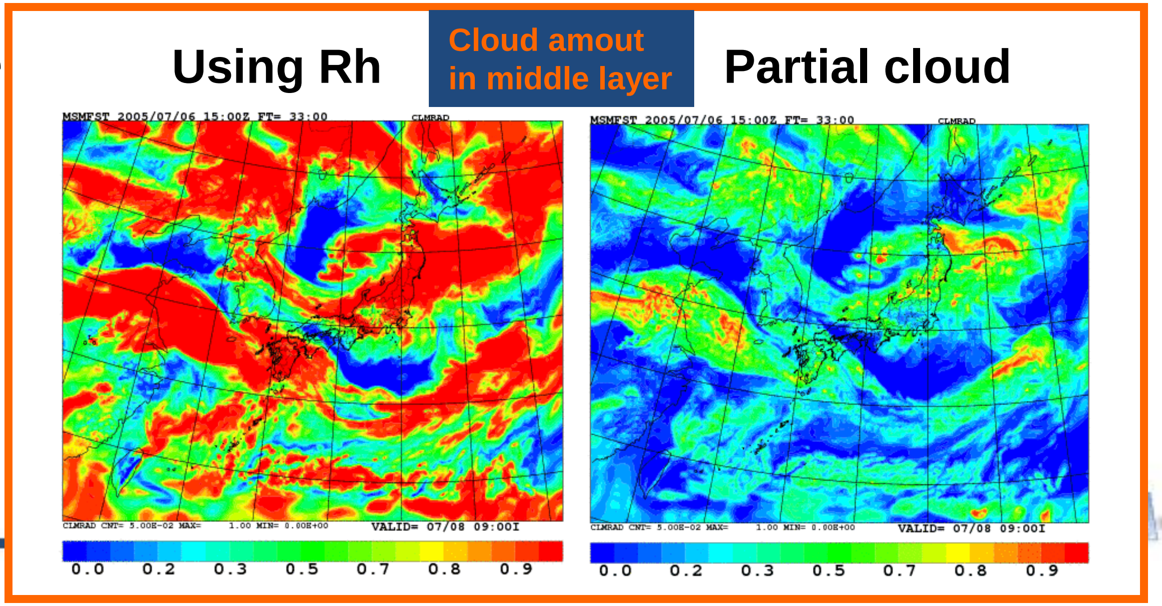
- Clear sky radiation scheme
  - Short wave : K-distribution method, 22bands
  - Long wave : table reference method and K-distribution method, 9bands
  - Consider direct effects by aerosol
  - Use three dimensional Ozone climatological data

- Cloud radiation scheme
  - Short wave
    - Calculate optical characteristics of cloud by cloud water / ice and radius of cloud particle
    - depends on the band
  - Long wave
    - Calculate the effect of cloud which cannot assume black body
    - Cloud amount is corrected by emiting ratio.
    - no dependency on bands
  - Cloud amount and cloud water are diagnosed by partial condensation scheme in MYNN3 (No use of cloud water and ice by cloud physics)
  - Using maximum-random overlap

# Partial condensation scheme

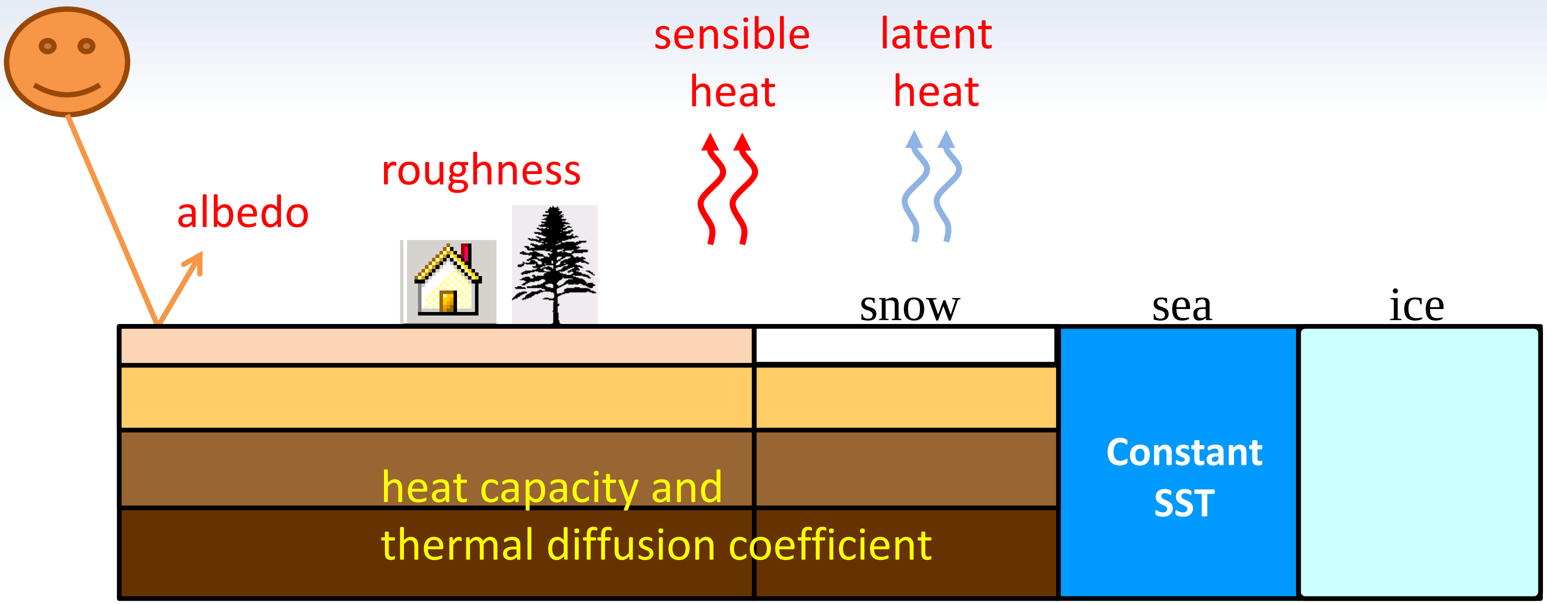
- **Consider condensation of subgrid scale**
  - Express perturbations of moisture and potential temperature in each grid using Probability Distribution Function (PDF).
- **Calculate TKE production term**
  - buoyancy flux
  - for turbulent scheme




気象庁
Japan Meteorological Agency

# Surface Process (Flow Independent)

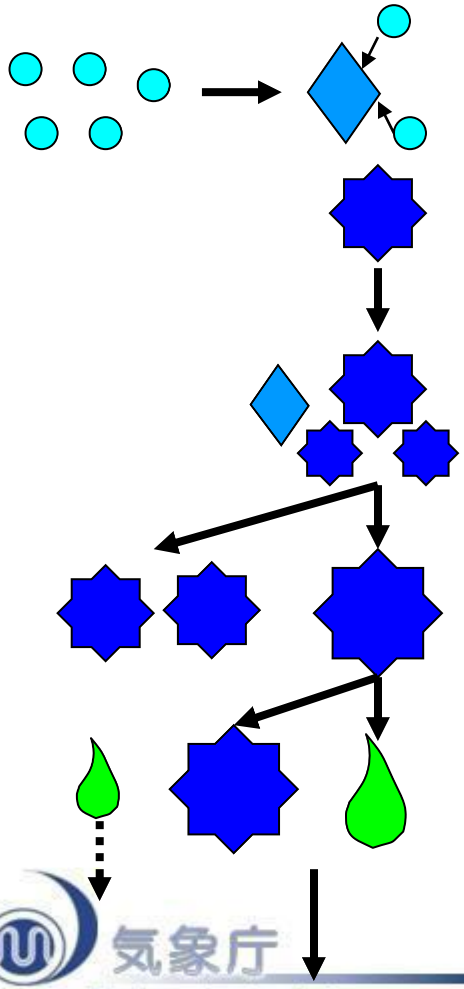



- Calculate surface fluxes
  - sensible / latent heat flux
  - upward short / long wave radiation flux

# Surface Process in JMA-NHM

- **Calculate surface fluxes to the atmosphere**
  - Give boundary condition for MYNN3
  - Formulation : Monin-Obukhov similarity theory
  - Gradient functions : Beljaars and Holtslag(1991)
- **Estimate the surface temperature**
  - To calculate surface fluxes
  - 4-layer heat conduction equation
- **Forecast the soil moisture**
  - To estimate evapolation efficiency at the surface
  - Force-restore method (Deardorff, 1978)

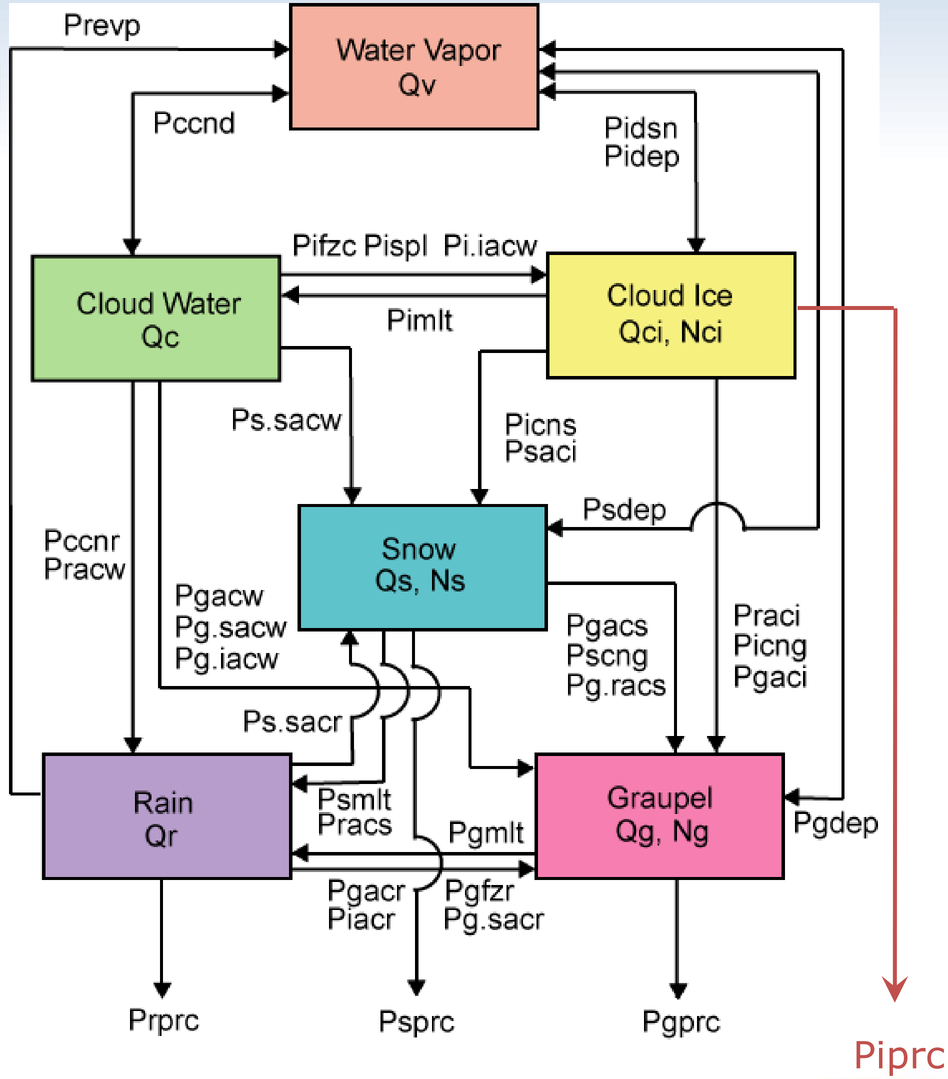
# Cloud Physics (Local Source or Sink)



- **Predicts the mixing ratios of water vapor and five hydrometeors**
  - cloud water, cloud ice, rain, snow, and graupel
  - The diabatic heat invloved a **change of phase** affects atmospheric temperature, and contributes updraft or downdraft.
    - The diabatic heat is calculated in “dynamics”.

# Cloud Physics in JMA-NHM

- Forecast mixing ratio and number concentration
- Processes
  - Ice particle production process
  - Condensation growth
  - Evaporative growth
  - Sublimation growth
  - Diffusive growth
  - Crash and merge growth
  - etc…



Introduction of JMA-NHM

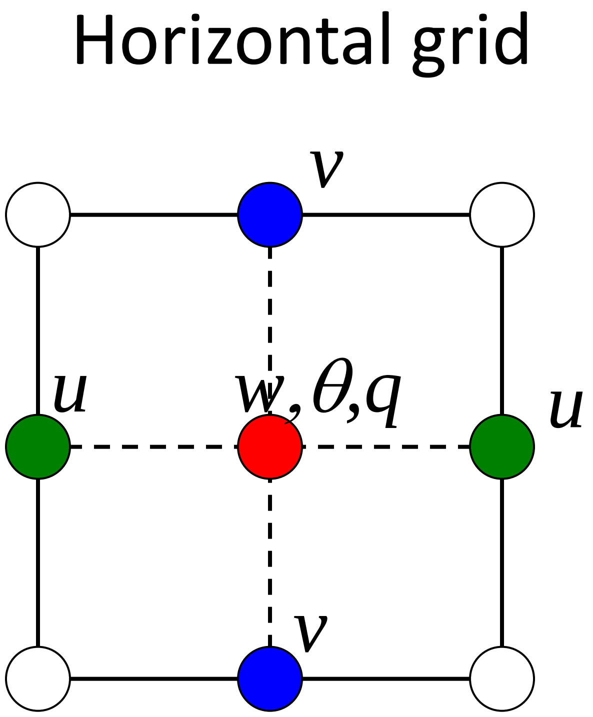
# GRID STRUCTURE

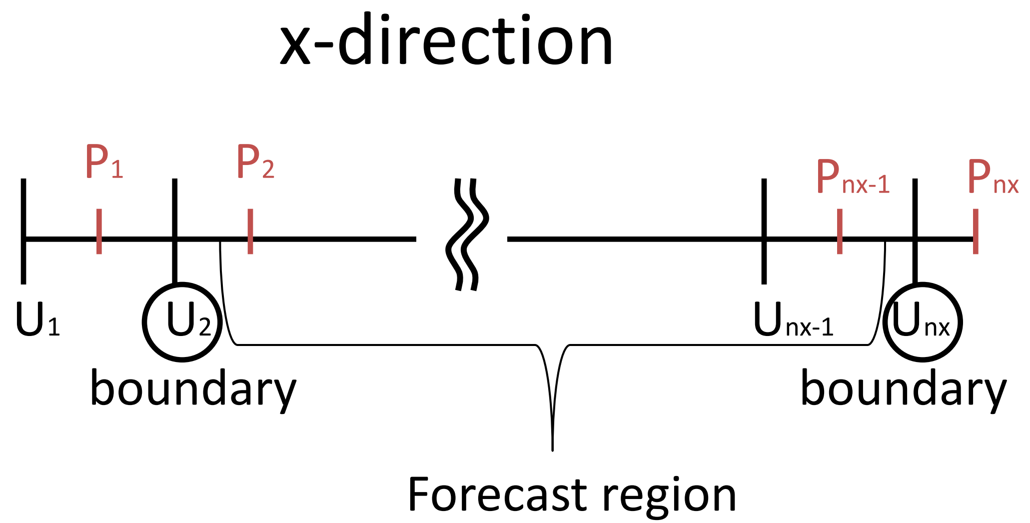
# Grid Structure

- Horizontal grid
- Vertical grid

# Horizontal Grid (Arakawa-C grid)

Advantage for horizontal advection



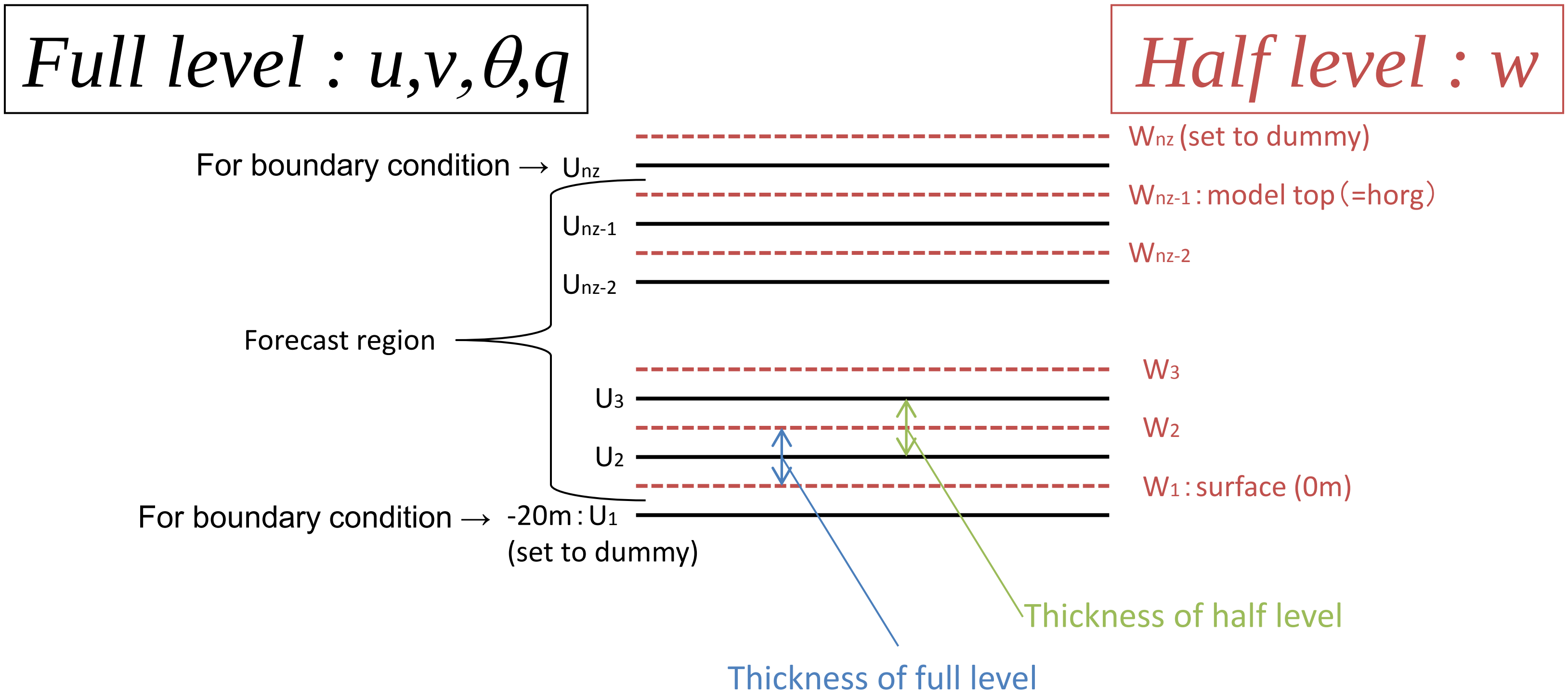




$U_{ix}$ : Vector point

$P_{ix}$ : Scalar point

# Vertical Grid (Lorenz Grid)

Advantage for vertical advection




In the code

- zrp : vertical coordinate value at full level
- zrw : vertical coordinate value at half level
- vdz : Thickness of full level (its inverse : vrdz)
- vdz2 : Thickness of half level

Introduction of JMA-NHM

# 5. PROCESSES IN ONE TIMESTEP OF JMA-NHM

# Processes in One Timestep of JMA-NHM

- **Caution!**
  - The following slides show **original processes of JMA-NHM**
    - Different from other models

# Forecasting Equations

## Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{g p'}{C_m^2} - \mathrm{buoy}\right) \delta_3^i + 2\rho \epsilon^{ijk} \Omega_j u_k = \rho \mathrm{Dif}.u^i$$

## Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ -\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m} \frac{\partial \theta_m}{\partial t} \right\}$$

## Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho} \left\{ \nabla_i (\rho u^i \theta) - \theta \nabla_i (\rho u^i) \right\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

## Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho} \left\{ \nabla_i (\rho u^i q_n) - q_n \nabla_i (\rho u^i) \right\} = Q_n + \mathrm{Dif}.q_n$$

# Processes

## Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{g p'}{C_m^2} - \mathrm{buoy}\right)\delta_3^i + 2\rho\epsilon^{ijk}\Omega_j u_k = \rho \mathrm{Dif}.u^i$$

**1. Advection** **1. Coriolis**

## Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ -\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m}\frac{\partial \theta_m}{\partial t} \right\}$$

## Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho}\left\{\nabla_i(\rho u^i \theta) - \theta \nabla_i(\rho u^i)\right\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

## Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho}\left\{\nabla_i(\rho u^i q_n) - q_n \nabla_i(\rho u^i)\right\} = Q_n + \mathrm{Dif}.q_n$$

# Processes

Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{gp'}{C_m^2} - \mathrm{buoy}\right)\delta_3^i + 2\rho\epsilon^{ijk}\Omega_j u_k = \rho \mathrm{Dif}.u^i$$

Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ -\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m}\frac{\partial \theta_m}{\partial t} \right\}$$

Thermodynamic equation

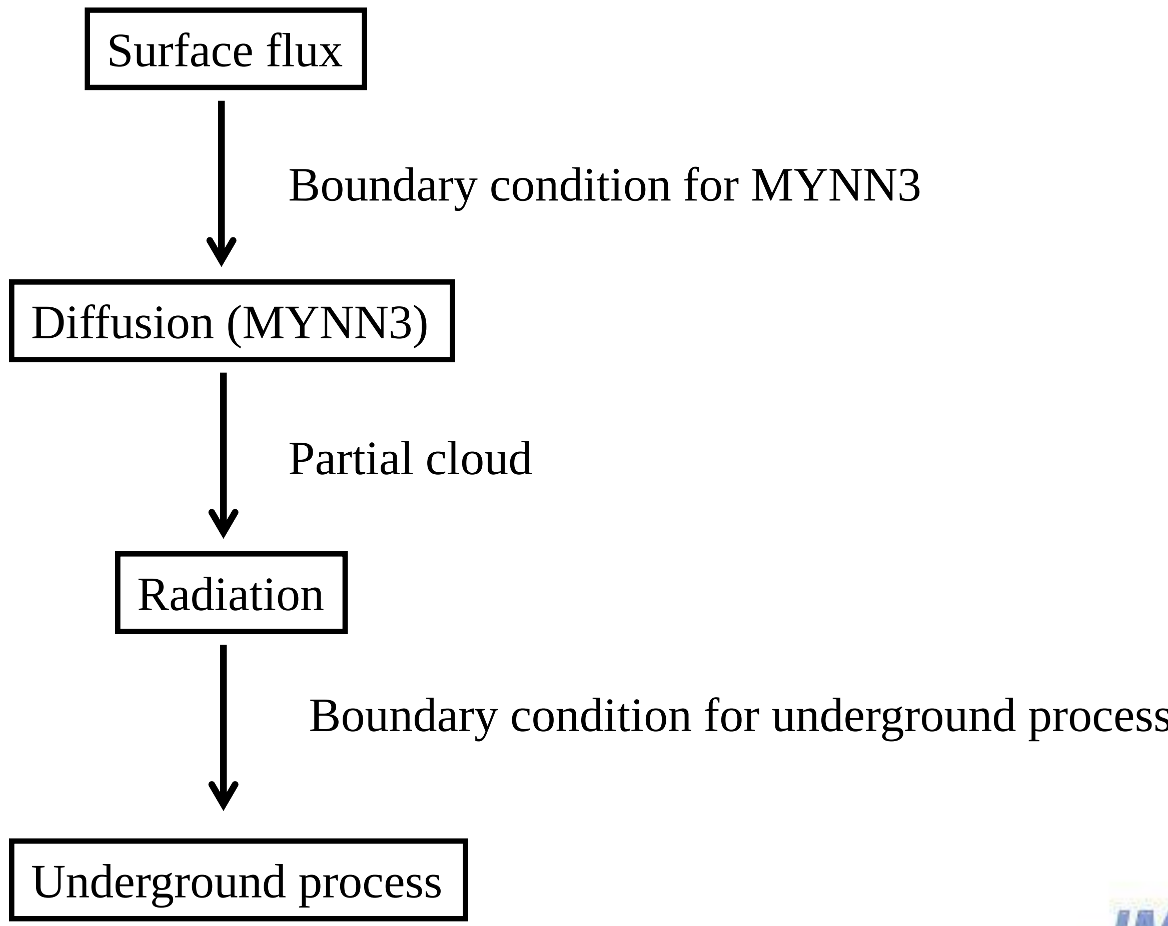
**2. Surface flux, turbulence, and diabatic process**

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho}\{\nabla_i(\rho u^i \theta) - \theta \nabla_i(\rho u^i)\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho}\{\nabla_i(\rho u^i q_n) - q_n \nabla_i(\rho u^i)\} = Q_n + \mathrm{Dif}.q_n$$

# Surface Flux, Diffusion, and Diabatic Process



# Processes

Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{g p'}{C_m^2} - \mathrm{buoy}\right) \delta_3^i + 2\rho\epsilon^{ijk}\Omega_j u_k = \rho \mathrm{Dif}.u^i$$

Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ -\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m} \frac{\partial \theta_m}{\partial t} \right\}$$

Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho} \left\{ \nabla_i (\rho u^i \theta) - \theta \nabla_i (\rho u^i) \right\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

Water substances

**3. Cloud physics and KF scheme**

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho} \left\{ \nabla_i (\rho u^i q_n) - q_n \nabla_i (\rho u^i) \right\} = Q_n + \mathrm{Dif}.q_n$$

# Processes

Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{gp'}{C_m^2} - \mathrm{buoy}\right)\delta_3^i + 2\rho\epsilon^{ijk}\Omega_j u_k = \rho \mathrm{Dif}.u^i$$

**4. Diagnose buoyancy term**

Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{-\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m}\frac{\partial \theta_m}{\partial t}\right\}$$

Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho}\left\{\nabla_i(\rho u^i \theta) - \theta \nabla_i(\rho u^i)\right\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho}\left\{\nabla_i(\rho u^i q_n) - q_n \nabla_i(\rho u^i)\right\} = Q_n + \mathrm{Dif}.q_n$$

# Processes

**Momentum equations**

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{g p'}{C_m^2} - \mathrm{buoy}\right) \delta_3^i + 2\rho\epsilon^{ijk}\Omega_j u_k = \rho \mathrm{Dif}.u^i$$

**Pressure equation**

**5. HE-VI**

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ -\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m} \frac{\partial \theta_m}{\partial t} \right\}$$

**Thermodynamic equation**

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho} \{\nabla_i(\rho u^i \theta) - \theta \nabla_i(\rho u^i)\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

**Water substances**

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho} \{\nabla_i(\rho u^i q_n) - q_n \nabla_i(\rho u^i)\} = Q_n + \mathrm{Dif}.q_n$$

# Processes

**Complete one timestep !**

## Momentum equations

$$\frac{\partial(\rho u^i)}{\partial t} + \nabla_j \cdot (\rho u^i u^j) - u^i \mathrm{prc} + (\nabla p')^i + \left(\sigma \frac{g p'}{C_m^2} - \mathrm{buoy}\right) \delta_3^i + 2\rho \epsilon^{ijk} \Omega_j u_k = \rho \mathrm{Dif}.u^i$$

## Pressure equation

$$\frac{\partial p}{\partial t} = C_m^2 \left\{ -\nabla_i \cdot (\rho u^i) + \mathrm{prc} + \frac{\rho}{\theta_m} \frac{\partial \theta_m}{\partial t} \right\}$$

## Thermodynamic equation

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho} \left\{ \nabla_i (\rho u^i \theta) - \theta \nabla_i (\rho u^i) \right\} = \frac{Q}{C_p \pi} + \mathrm{Dif}.\theta$$

## Water substances

$$\frac{\partial q_n}{\partial t} + \frac{1}{\rho} \left\{ \nabla_i (\rho u^i q_n) - q_n \nabla_i (\rho u^i) \right\} = Q_n + \mathrm{Dif}.q_n$$

# If you want to know more details about the NHM, See below references...

- **References:**

  - Saito, K., T. Fujita, Y. Yamada, J. Ishida, Y. Kumagai, K. Aranami, S. Ohmori, R. Nagasawa, S. Kumagai, C. Muroi, T. Kato, H. Eito, and Y. Yamazaki, 2006: The operational JMA nonhydrostatic model. *Mon. Wea. Rev*., **134,** 1236-1298.
  - Saito, K., J. Ishida, K. Aranami, T. Hara, T. Segawa, M. Narita and Y. Honda, 2007: Nonhydrostatic atmospheric models and operational development at JMA. J. Meteor. Soc. Japan, 85B, 271-304.
  - Kazuo Saito, Teruyuki Kato, Hisaki Eito and Chiashi Muroi, 2001: Documentation of the Meteorological Research Institute / Numerical Prediction Division Unified Nonhydrostatic Model. http://www.mri-jma.go.jp/Publish/Technical/DATA/VOL_42/42_en.html
  - Motohki Ikawa, Kazuo Saito, 1991: Description of a Nonhydrostatic Model Developed at the Forecast Research Department of the MRI. http://www.mri-jma.go.jp/Publish/Technical/DATA/VOL_28/28_en.html


END