

Introduction and Recent Advances of Data Assimilation

Masaru Kunii

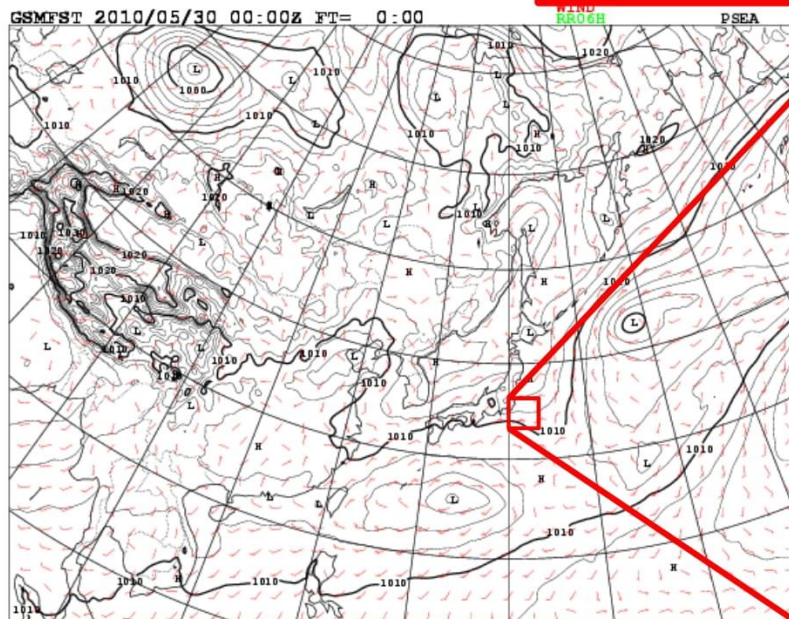
Meteorological Research Institute / JMA, Japan

mkunii@mri-jma.go.jp

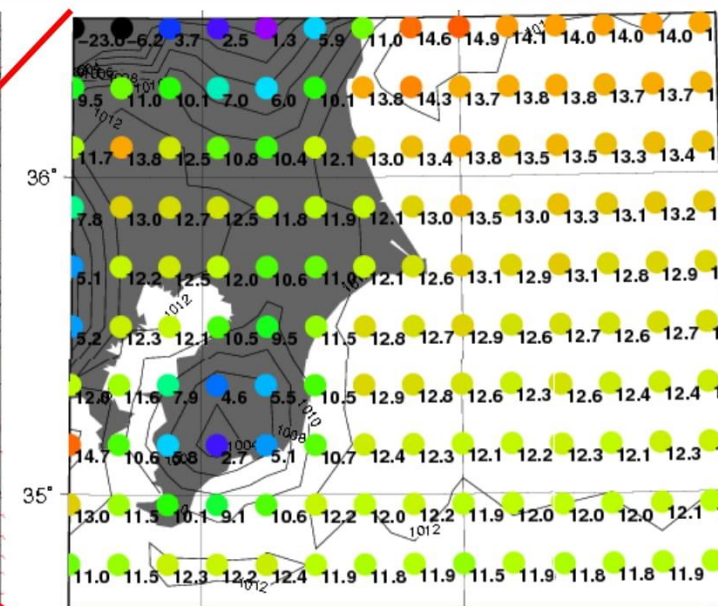
Objective Analysis

- Analyze atmospheric state \mathbf{x}_t at a given time t numerically. Here \mathbf{x}_t is a set of all the variables on all the grid points
 - $1,312,360_{(\text{Number of horizontal grids})} \times (1_{(P)} + 60_{(\text{Layer})} \times 4_{(\text{UVTQ})}) \approx 316\text{e6}$

How determine this huge amount of data, accurately ?



P_{MSL} (Mean Sea Level)
Weather map around Japan



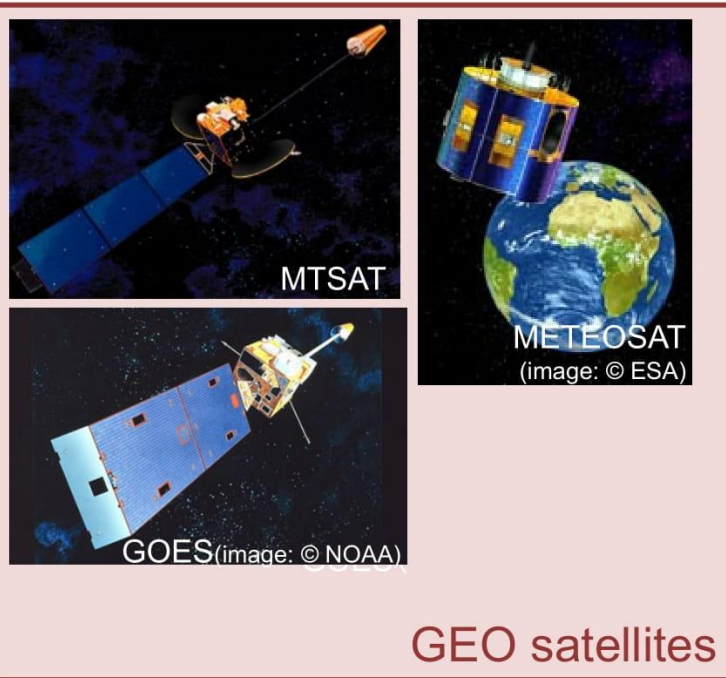
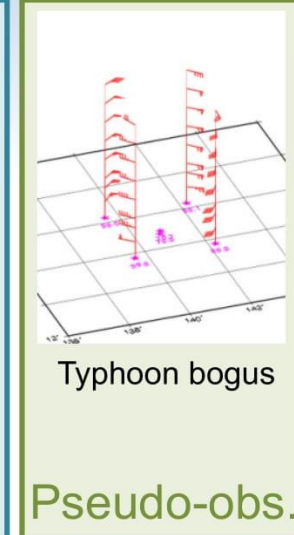
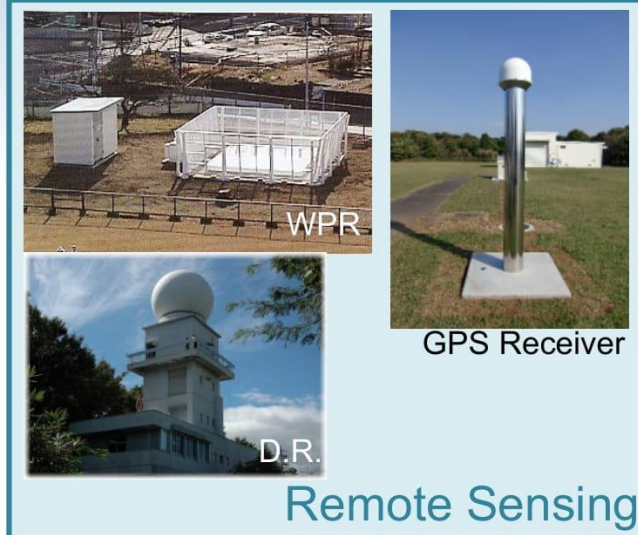
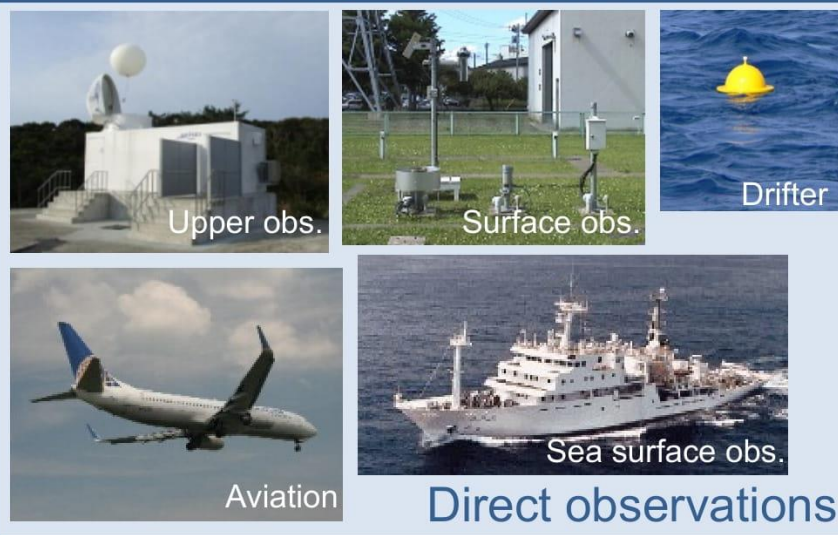
$P_{\text{S-1000}}$
Original GPV distribution
many values are included in this small box.



Available data for objective analysis #1

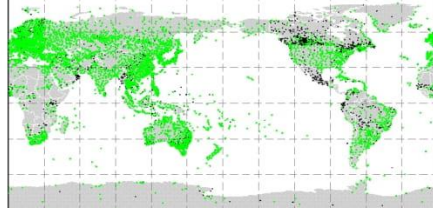
- A variety of observations
 - 😊: The data reflects real atmospheric state
 - 😊: while it includes observation error
 - 😞: All the variables on all the grid points are NOT available

Observations assimilated in NWP



Data coverage (Global cycle analysis)

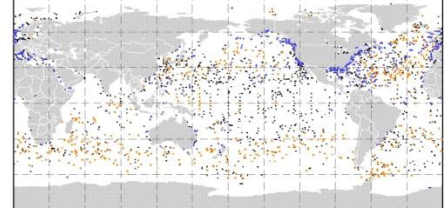
LAND SURFACE 2011/03/28 00:00(UTC)



SYNOPS 3368
NOUSET 11551
ALL 15528

METAR
NOUSET 3613
ALL 3613

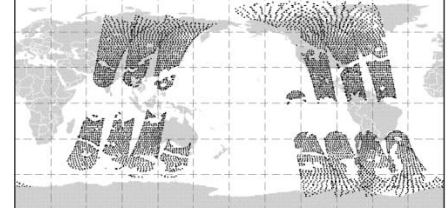
SEA SURFACE 2011/03/28 00:00(UTC)



SHIP 404
NOUSET 3062
ALL 3466

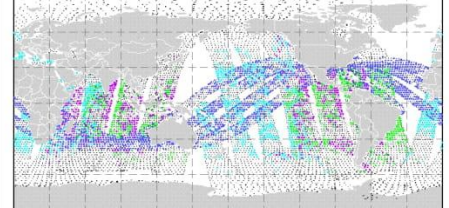
DRIFTER 541
NOUSET 8374
ALL 8915

MW-SOUNDER(SSMIS) 2011/03/28 00:00(UTC)



DMSF-F16
NOUSET 6121
ALL 6121

MW-IMAGER 2011/03/28 00:00(UTC)



DMSF-F16
NOUSET 2384
ALL 2384

DMSF-F17
NOUSET 2094
ALL 2094

TRMM
NOUSET 1295
ALL 1295

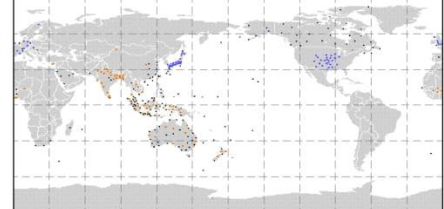
TRMM
NOUSET 5448
ALL 5448

UPPER(TEMP) 2011/03/28 00:00(UTC)



TEMP 667
NOUSET 5
ALL 672

UPPER(PILOT/WPROF) 2011/03/28 00:00(UTC)



PILOT 25
NOUSET 144
ALL 171

WPROF 68
NOUSET 134
ALL 1602

MW-SOUNDER(AMSU-A) 2011/03/28 00:00(UTC)



NOAA-15
NOUSET 4134
ALL 4134

NOAA-16
NOUSET 4358
ALL 4358

NOAA-18
NOUSET 5515
ALL 5515

NOAA-19
NOUSET 2753
ALL 2753

NOAA-19
NOUSET 112
ALL 112

NOAA-19
NOUSET 4867
ALL 4867

NOAA-19
NOUSET 112
ALL 112

NOAA-15
NOUSET 5139
ALL 5139

NOAA-16
NOUSET 5689
ALL 5689

NOAA-18
NOUSET 1581
ALL 1581

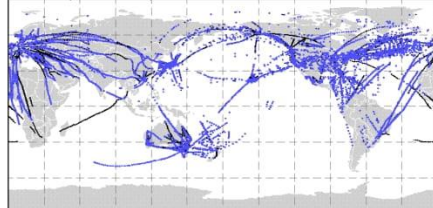
NOAA-19
NOUSET 1332
ALL 1332

NOAA-19
NOUSET 4636
ALL 4636

NOAA-19
NOUSET 2555
ALL 2555

NOAA-19
NOUSET 4336
ALL 4336

UPPER(AVIATION)/BOGUS 2011/03/28 00:00(UTC)

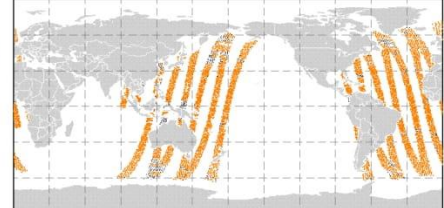


TYBOGUS 0
NOUSET 0
ALL 0

YHTC
NOUSET 0
ALL 0

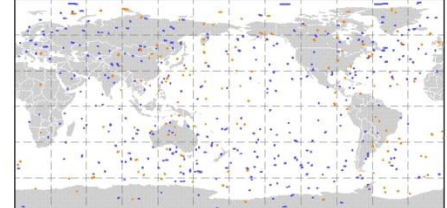
AVIATION 1537
NOUSET 26347
ALL 31884

SCATTEROMETER 2011/03/28 00:00(UTC)



METOP-2
NOUSET 4294
ALL 5573

GNSS RADIO OCCULTATION 2011/03/28 00:00(UTC)



METOP-2
NOUSET 2384
ALL 2384

ASCAT 4294
NOUSET 5573
ALL 5573

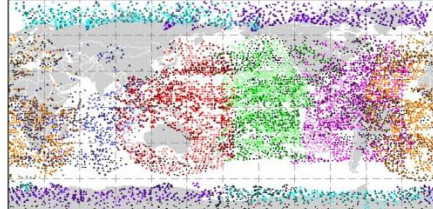
ASCAT 4294
NOUSET 5573
ALL 5573

ASCAT 4294
NOUSET 5573
ALL 5573

ASCAT 4294
NOUSET 5573
ALL 5573

ASCAT 4294
NOUSET 5573
ALL 5573

ATMOSPHERIC MOTION VECTOR 2011/03/28 00:00(UTC)



MTSAT-2
NOUSET 406
ALL 406

GOES-11
NOUSET 522
ALL 522

GOES-13
NOUSET 139
ALL 139

Meteosat-7
NOUSET 193
ALL 193

Meteosat-8
NOUSET 193
ALL 193

Terra
NOUSET 326
ALL 326

Aqua
NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

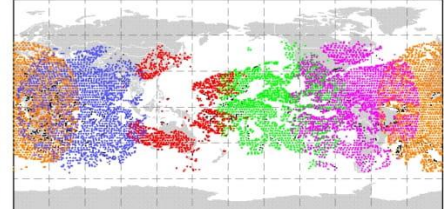
NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

NOUSET 336
ALL 336

CLEAR SKY RADIANCE 2011/03/28 00:00(UTC)



MTSAT-2
NOUSET 1180
ALL 1180

GOES-11
NOUSET 2000
ALL 2000

GOES-13
NOUSET 3373
ALL 3373

Meteosat-7
NOUSET 2907
ALL 2907

Meteosat-8
NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

NOUSET 607
ALL 607

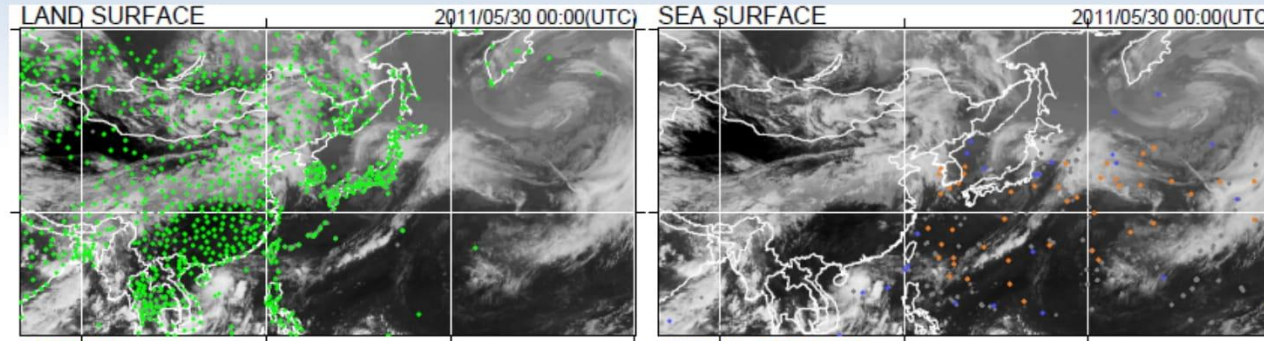
NOUSET 607
ALL 607

NOUSET 607
ALL 607

Data coverage with IR image #1/2

SYNOPSIS

Land surface only

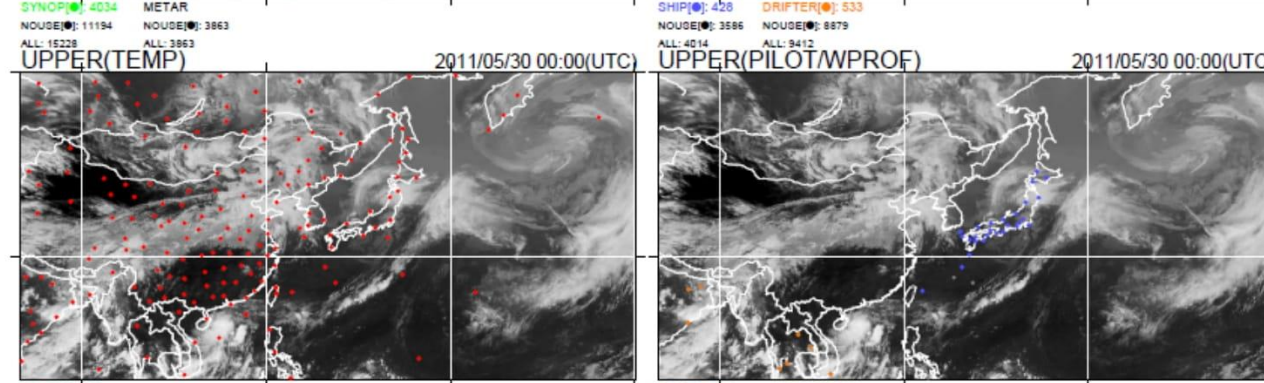


SHIP/BUOY

Sea Surface only, sparse

TEMP

Mostly over land area

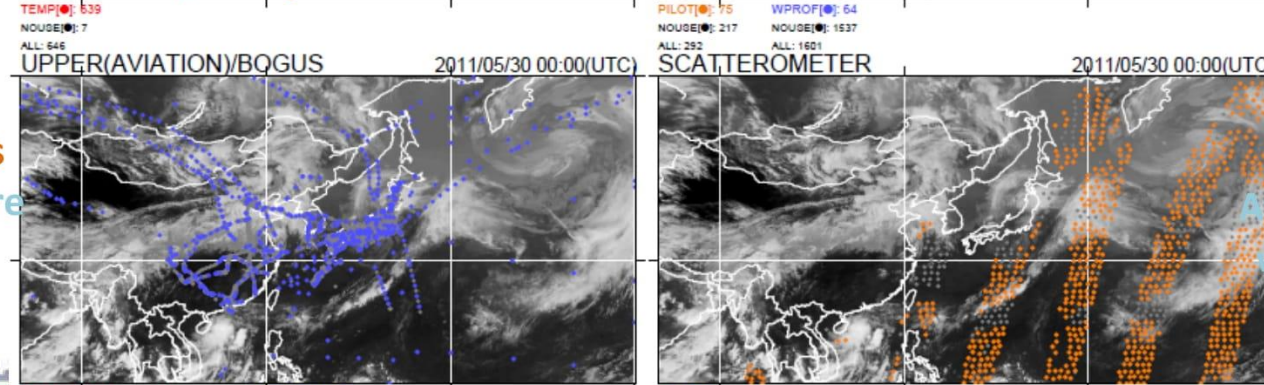


WPRF/PILOT

Only over land area

Aviation/ Typhoon bogus

Flight passes are mostly fixed



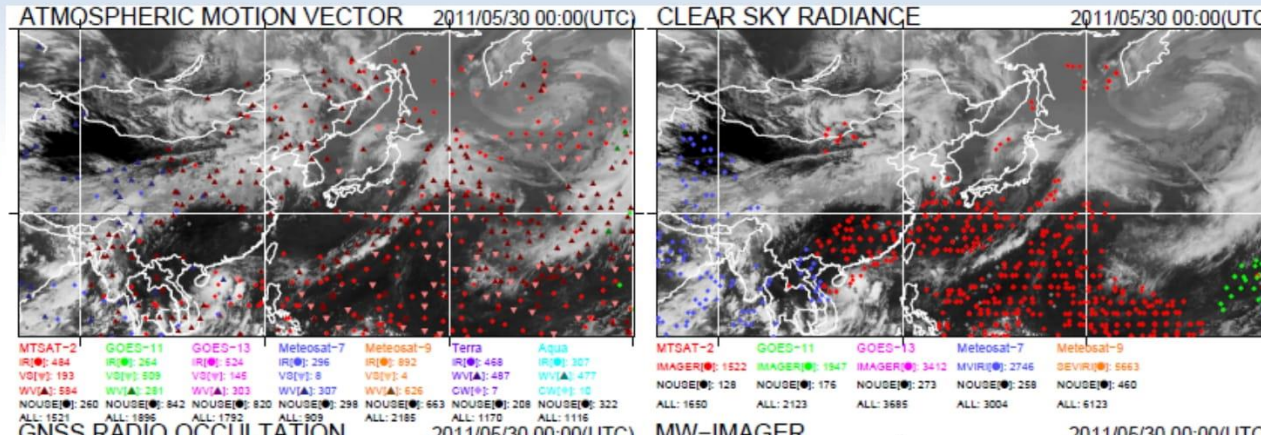
MW scatterometer (Sea surface wind)

Available over ocean with moderate wind

Data coverage with IR image #2/2

AMV

Cannot be produced from thick cloud

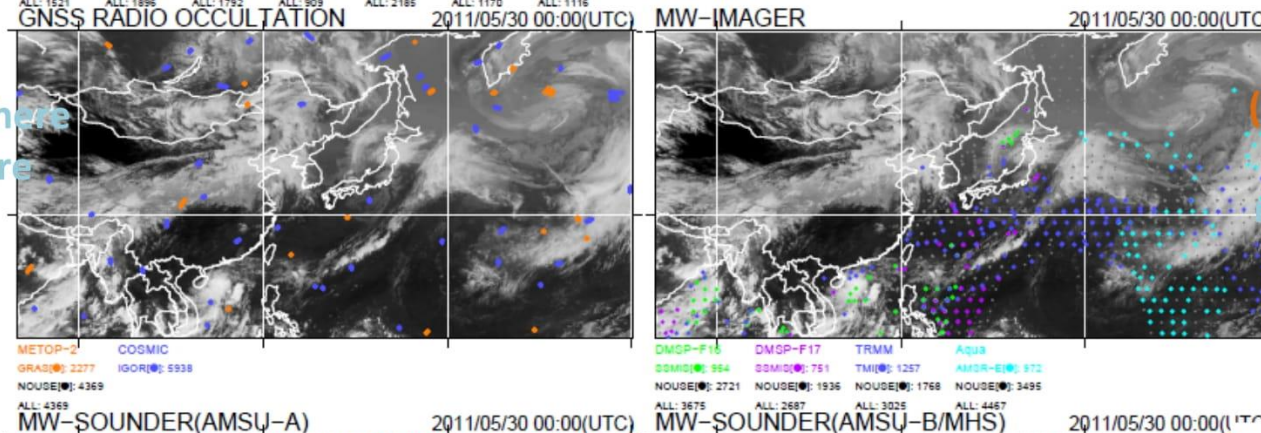


GEO-CSR

Upper troposphere
Only over clear sky area

GPS-RO

Upper troposphere and stratosphere

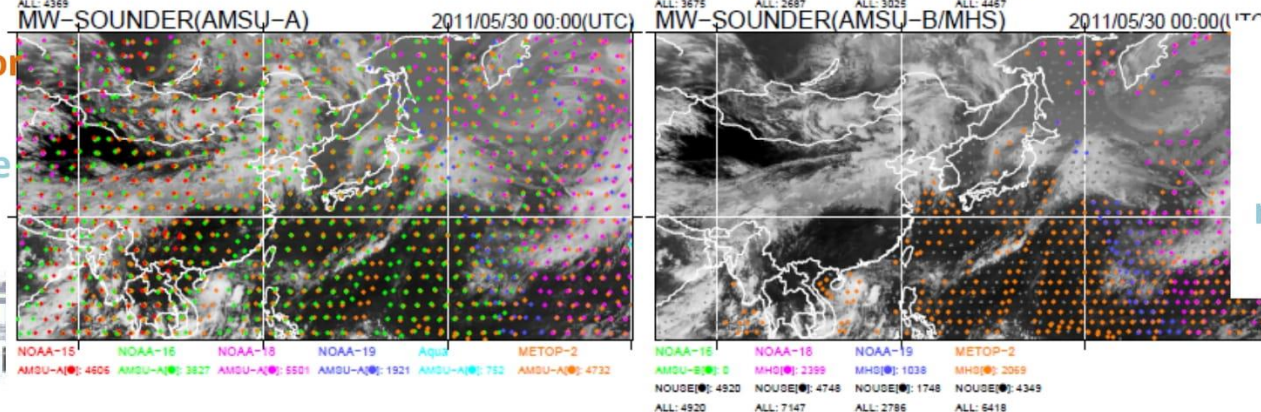


MW Imager

(Lower troposphere)
Only over ocean
Not assimilated over precipitation area

MW sounder for temperature

Depends on the atmospheric condition



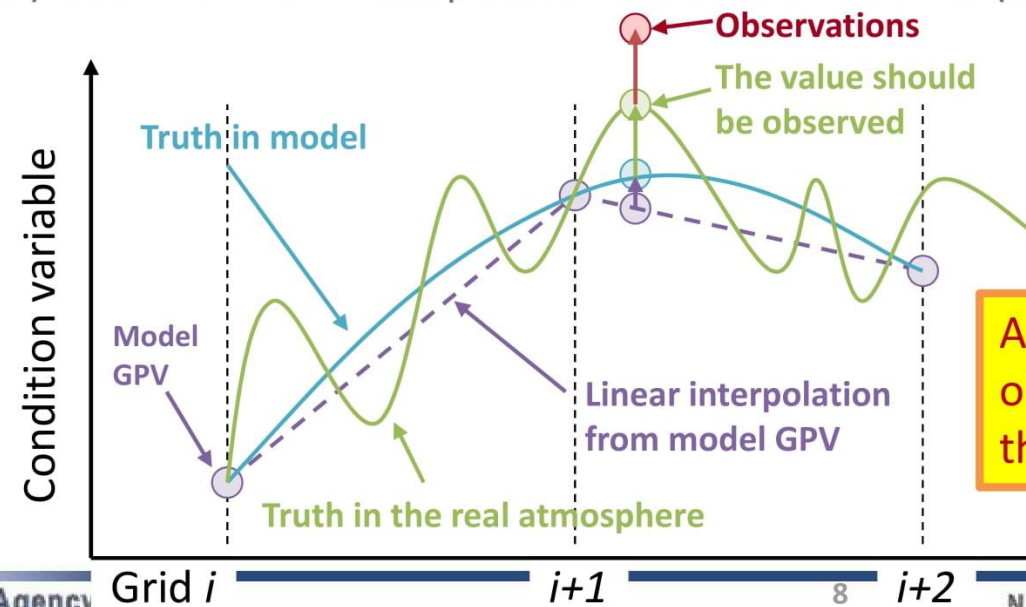
MW sounder for water vapor

Only over ocean
Not assimilated over precipitation area

Observation Error

- Observation error is composed by **measurement error**, **representative error**, and **conversion error**
 - **Measurement error**: The error from the instrument
 - **Representative error**: The error from spatial quantization
 - Perturbation from small scale phenomena is considered as “Error” ←
 - **Conversion error**: The error from observation operator
 - Ex) The error from interpolation method difference (wave/linear/cubic)

Since it cannot be represented in the NWP model

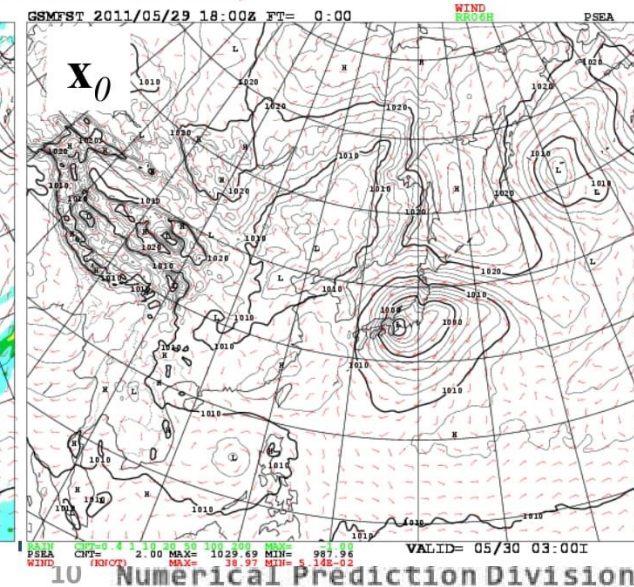
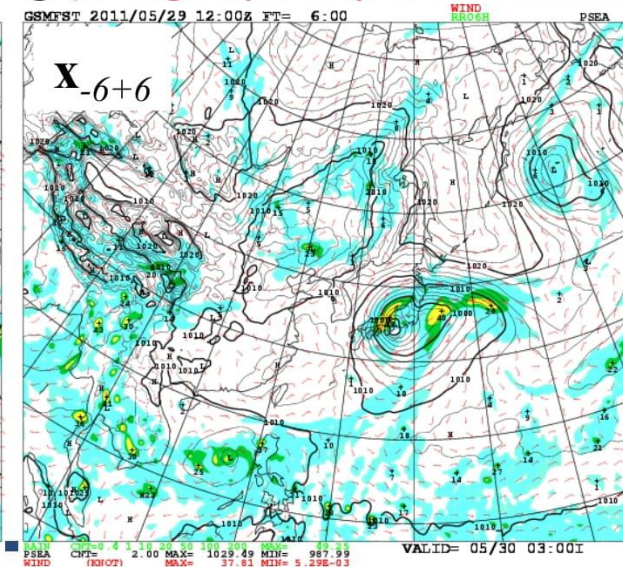
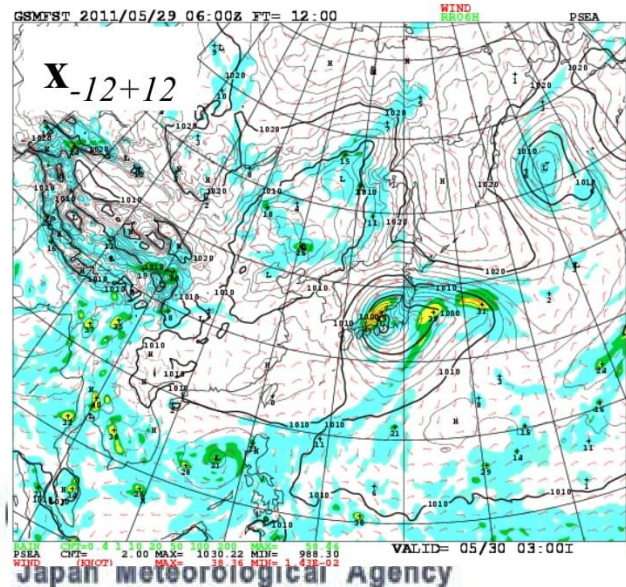


Available data for objective analysis #2

- NWP forecast GPV (the first guess)
 - ☺: All the variables on all the grid points are available
 - ☹: while it includes forecast error
 - ☹: It is NOT assured the forecast reflects real atmospheric state

Forecast values

- (Ex.) Forecast from the last analysis \mathbf{x}_{-dt+dt}
 - Background / First Guess
 - Ex. the case for global NWP system $dt=6[\text{hr}]$
 - 6-hours forecast from the analysis at 6-hours before
 - The forecast error should not be large if the forecast time was not so long (if high quality NWP model was used).



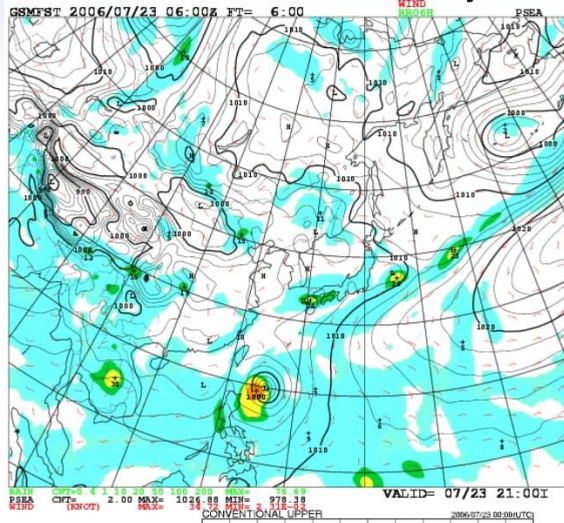
Data Assimilation (DA)

- Available data for objective analysis
 - A variety of observations
 - ☺: The data reflects real atmospheric state
 - » ☹: while it includes observation error
 - ☹: All the variables on all the grid points are NOT available
 - NWP forecast GPV (the first guess)
 - ☺: All the variables on all the grid points are available
 - » ☹: while it includes forecast error
 - ☹: It is NOT assured the forecast reflects real atmospheric state
- DA: Use the “☺” points from each data
 - Corrects the first guess with a variety of observations
 - “Assimilate a variety of observations with NWP GPV”

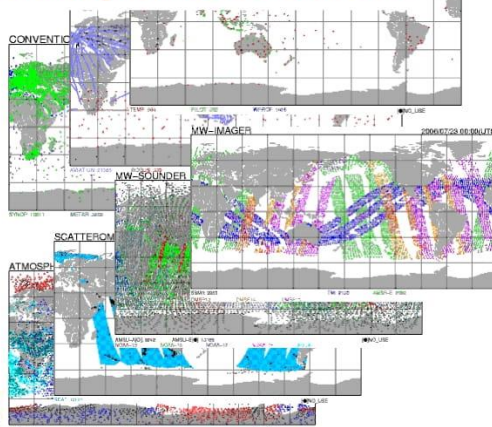
DA system

The first guess

Forecast from the last analysis

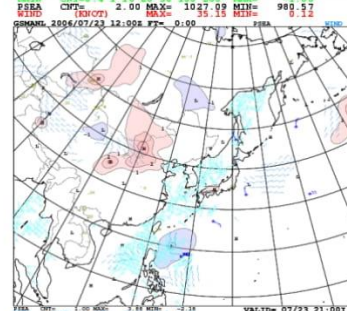
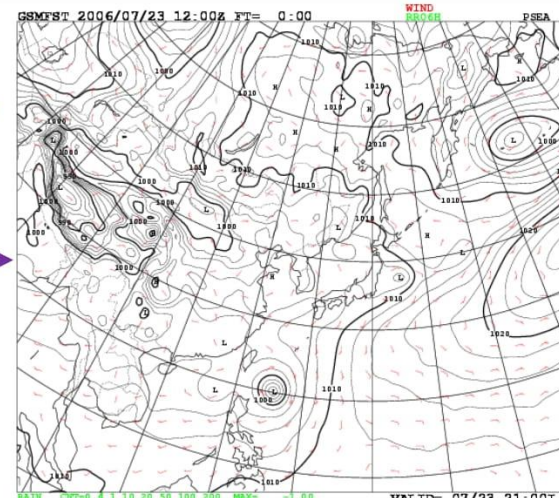


A variety of observations



DA

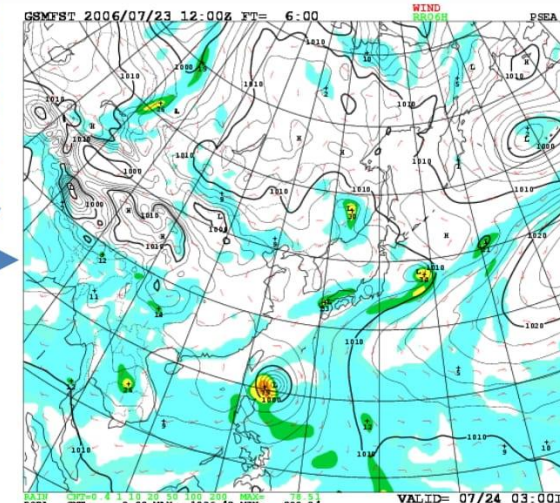
Analysis (Initial condition)



Increments

NWP

forecast from this analysis

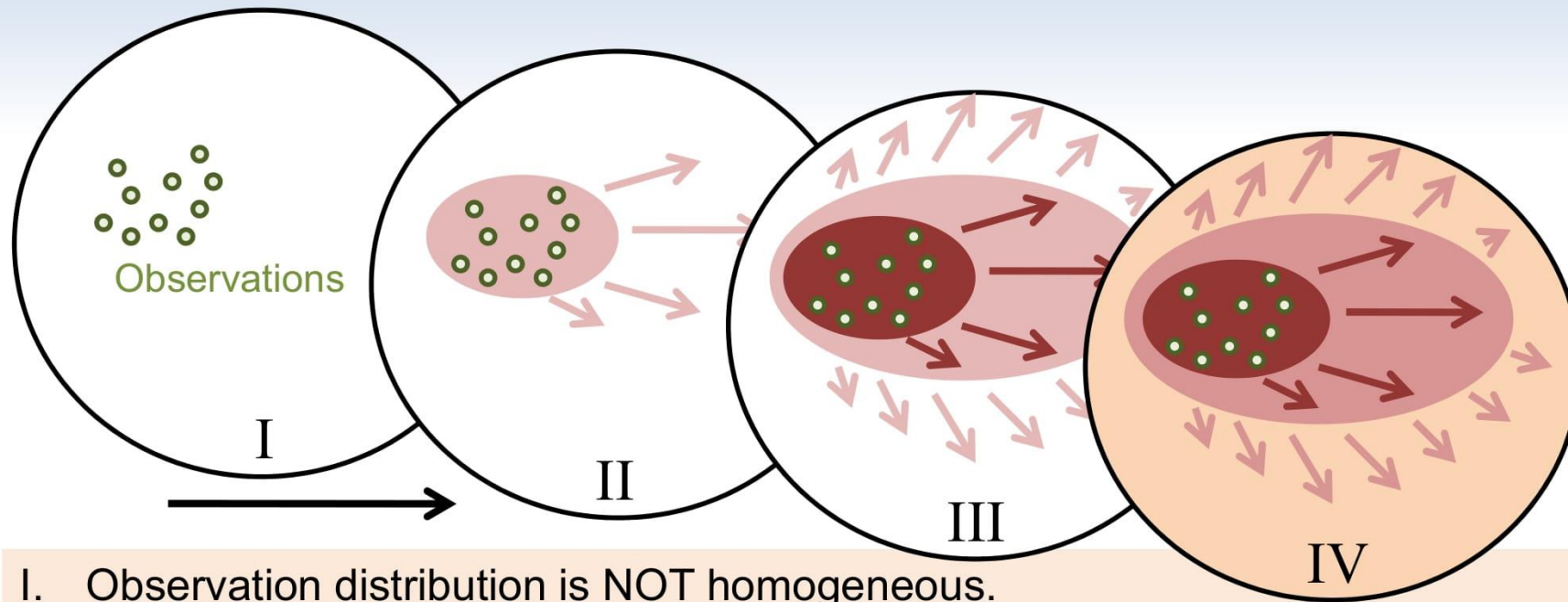


DA system:

The first guess is corrected by using a variety of observations. If the first guess was the forecast from the last analysis, this system would be operated cyclically.

➔ Analysis and forecast cycle

Analysis and forecast cycle



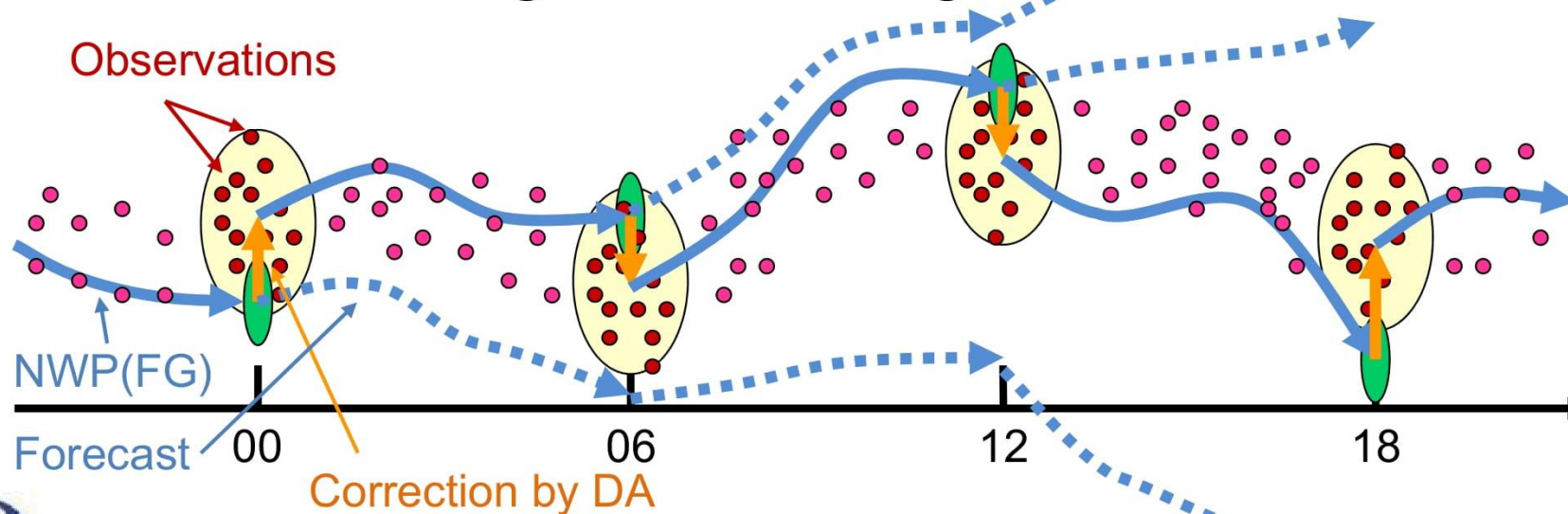
- I. Observation distribution is NOT homogeneous.
- II. The atmospheric state surrounding the observations is analyzed accurately. The well analyzed atmospheric state is transported by the atmospheric flow in the forecast to the external area.
- III. In the next DA, the atmospheric state surrounding the observations is analyzed more accurately. The better analyzed atmospheric state was transported in the next forecast more widely.
- IV. With this cyclic operation, the atmospheric state over the observation-poor area will be analyzed with the better accuracy.

Techniques for DA

- Two major approaches:
 - Minimum variance estimation
 - → Minimize the analysis error variance
 - Optimum Interpolation (OI) [used for surface analysis]
 - Ensemble Kalman filtering (EnKF)
 - Maximum likelihood estimation
 - → Find the maximum of likelihood function
 - \leftrightarrow Minimize the cost function
 - 3 dimensional variational method (3D-Var) [used for LA]
 - **4 dimensional variational method (4D-Var) [used for GA/MA]**

Analysis and forecast cycle

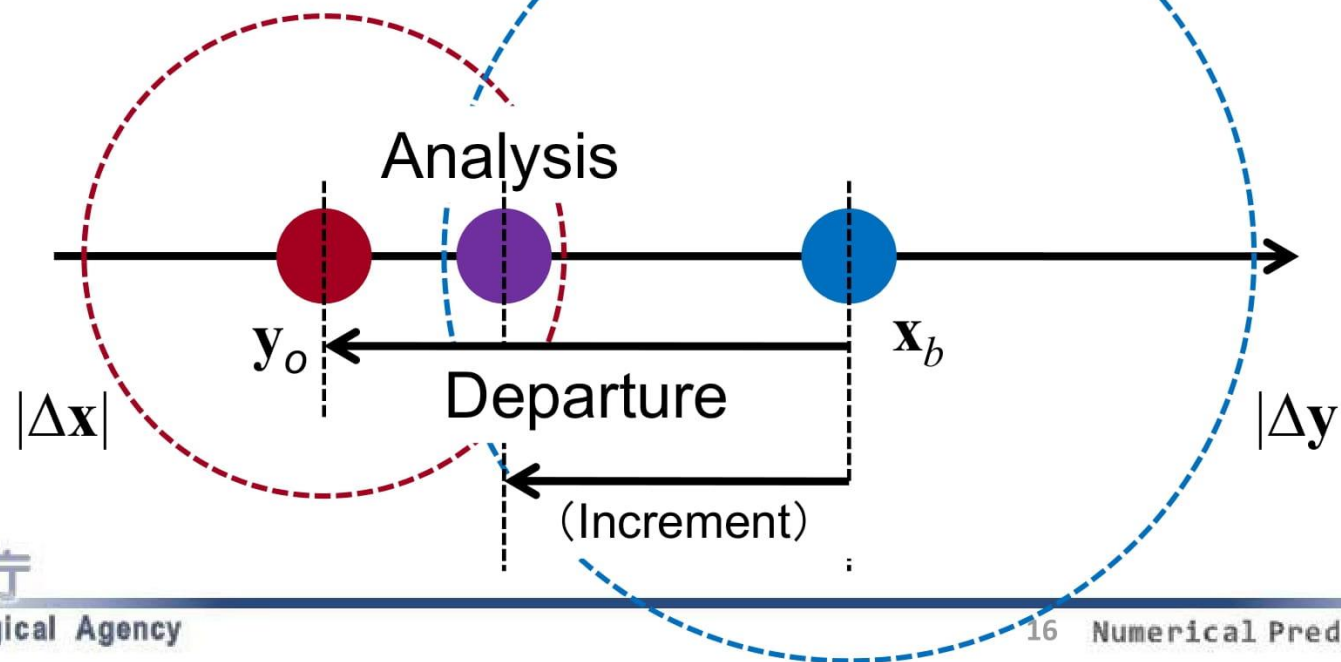
- Forecast error grows along forecast time
 - Short interval DA cycle maintains the analysis accuracy and the following forecast accuracy.
 - It is hard to correct the state in one analysis if the error of the first guess was too large



Determine the correction amount

(by using expected error information)

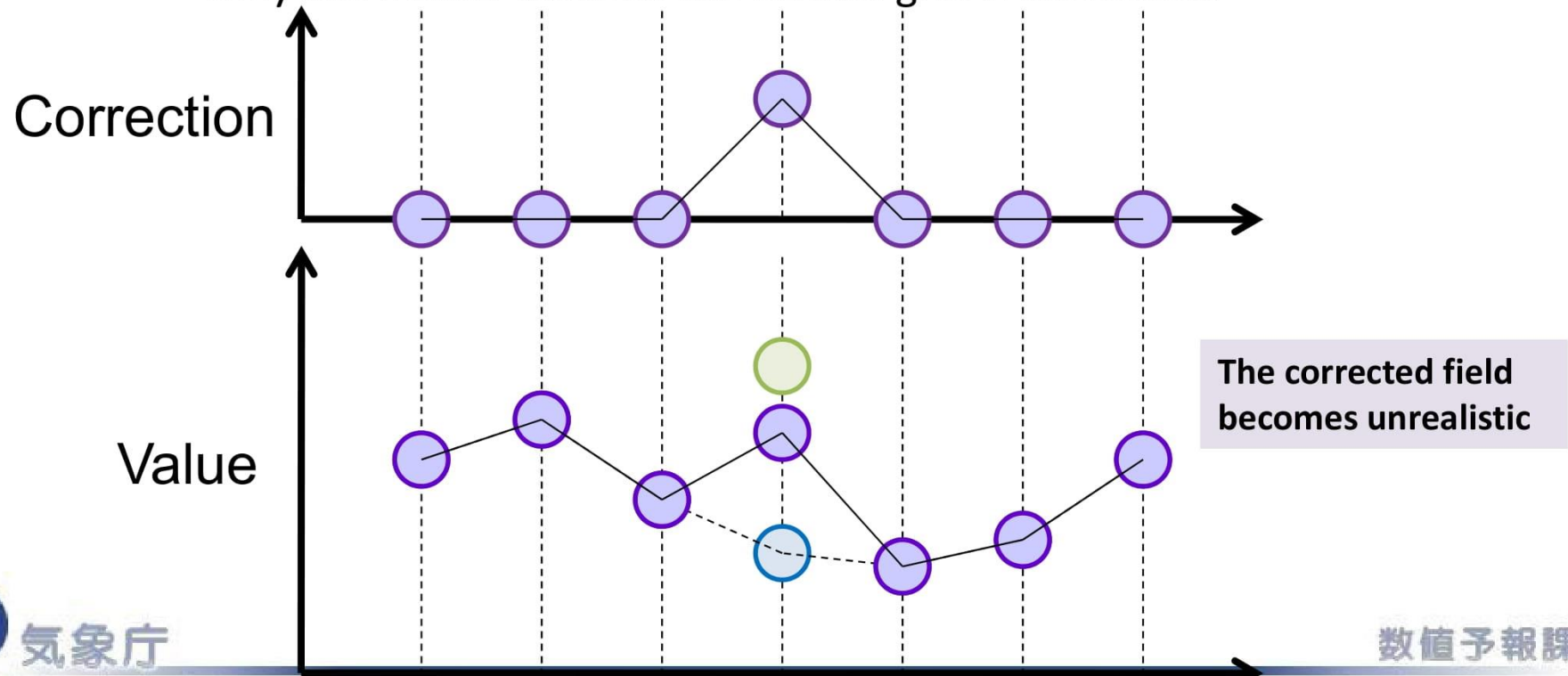
- Forecast (background) value \mathbf{x}_b contains error $\Delta\mathbf{x}$
- Observed value \mathbf{y}_o contains error $\Delta\mathbf{y}$
 - The expected errors $|\Delta\mathbf{x}|$ $|\Delta\mathbf{y}|$ can be estimated by statistics
 - With this information, the most likelihood value is estimated \rightarrow analyzed value \mathbf{x}_a



Correction for surrounding grids

Background error covariance

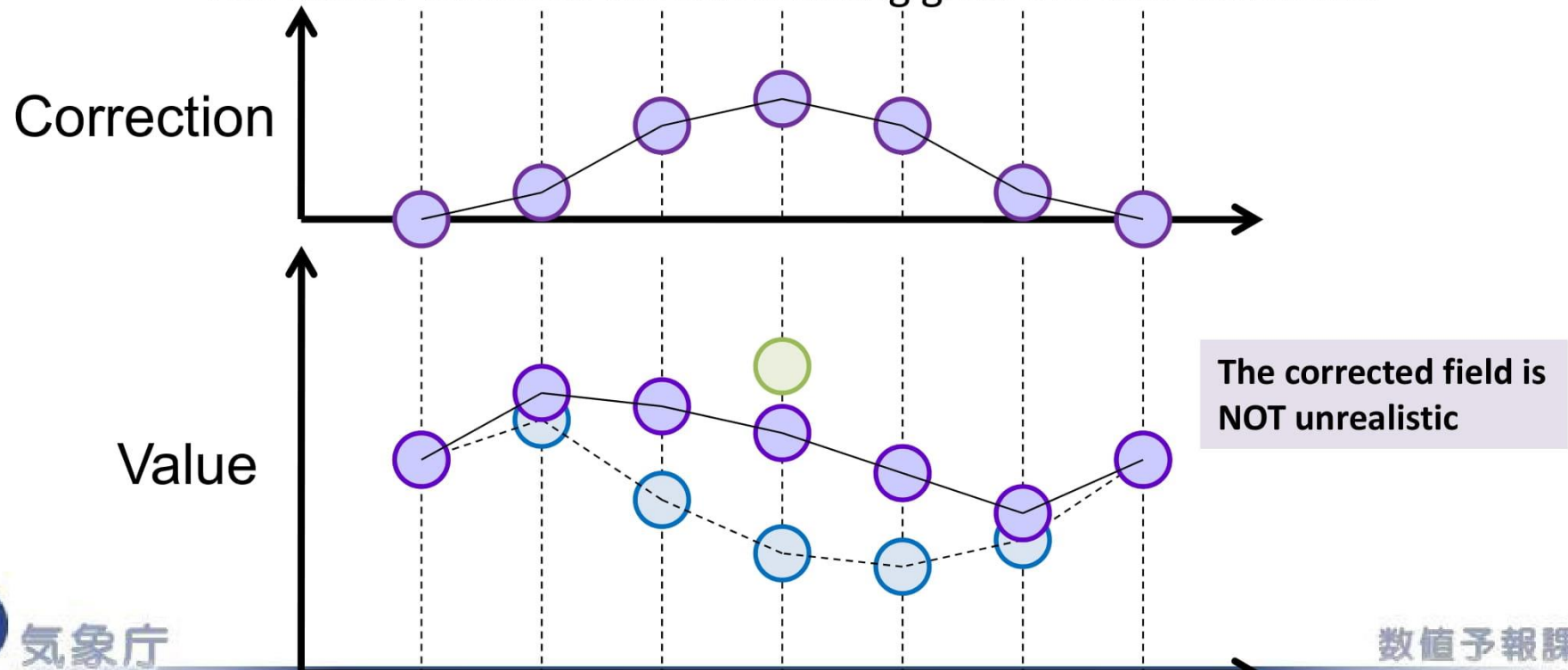
- When error was found in the model at a certain grid
 - The surrounding grids must have the resemble errors
 - If the background error covariance was NOT considered
 - Only the model value at the certain grid is corrected.



Correction for surrounding grids

Background error covariance

- When error was found in the model at a certain grid
 - The surrounding grids must have the resemble errors
 - If the background error covariance was considered
 - The model values at the surrounding grids are also corrected.



Variational method

- Define the cost function J as

$$2J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + (H\mathbf{x} - \mathbf{y}_o)^T \mathbf{R}^{-1} (H\mathbf{x} - \mathbf{y}_o)$$

- It is an index for the deviation of the analysis \mathbf{x}_a against the first guess \mathbf{x}_b and the one against observation \mathbf{y}_o .

- Size of \mathbf{x} is $O(8)$, Size of \mathbf{y} is $O(5)$
- \mathbf{B} : Background error covariance $\langle \Delta \mathbf{x}^T \Delta \mathbf{x} \rangle$
- \mathbf{R} : Observation error covariance $\langle \Delta \mathbf{y}^T \Delta \mathbf{y} \rangle$
- H : Observation operator

These matrices are simplified with various assumptions in the actual implementation (It is impossible to estimate explicitly)

- The operator calculates the value corresponding to the observation using GPV \mathbf{x}

- \mathbf{x}_a is estimated by non-linear optimization algorithm (iterative method) with using $\nabla_{\mathbf{x}} J$

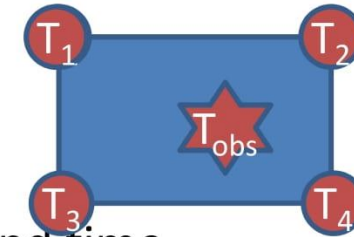
$$\nabla_{\mathbf{x}} J = \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + \mathbf{H}^T \mathbf{R}^{-1} (H\mathbf{x} - \mathbf{y}_o)$$

- \mathbf{H} : Tangent linear of H / \mathbf{H}^T : Adjoint operator (transpose of \mathbf{H})



Observation operator

- Observation operator
 - The operator to produce the value corresponding to the target observations from NWP GPVs.
 - Thus, it is an observation simulator
 - Simple example: Temperature (model variables)
 - Just an interpolation of GPVs for the location and time
 - Complicated example: Satellite radiance
 - Integrate the emission and absorption of radiance vertically



$$TB = \int B \frac{d\tau}{dz} dz + (1 - \varepsilon) \tau^2 \int \frac{B}{\tau^2} \frac{d\tau}{dz} dz + \tau_{srf} \varepsilon_{srf} B$$
$$= H(\mathbf{T}, \mathbf{q}, \nu, \theta, \mathbf{x}_{srf})$$

$$\because \tau = \tau(\nu, \theta, \mathbf{T}, \mathbf{q}), B = B(\nu, T), \varepsilon = \varepsilon(\nu, \theta, \mathbf{x}_{srf})$$

τ : Transmittance
 ε : Emissivity
 B : Plank function
 θ : Irradiance angle
 ν : Frequency
 \mathbf{T} : Temperature profile
 \mathbf{q} : Specific humidity profile
 \mathbf{x}_{srf} : Surface parameters

Tangent linear/Adjoint operator

- An example for wind speed

Observation operator

$$WS = H(\mathbf{x}) = H(u, v) = \sqrt{u^2 + v^2}$$

Tangent linear operator

$$\mathbf{H} = \begin{pmatrix} \frac{u}{\sqrt{u^2 + v^2}} & \frac{v}{\sqrt{u^2 + v^2}} \end{pmatrix}$$

$$WS\delta WS = u\delta u + v\delta v \leftarrow d(WS^2 = u^2 + v^2)$$

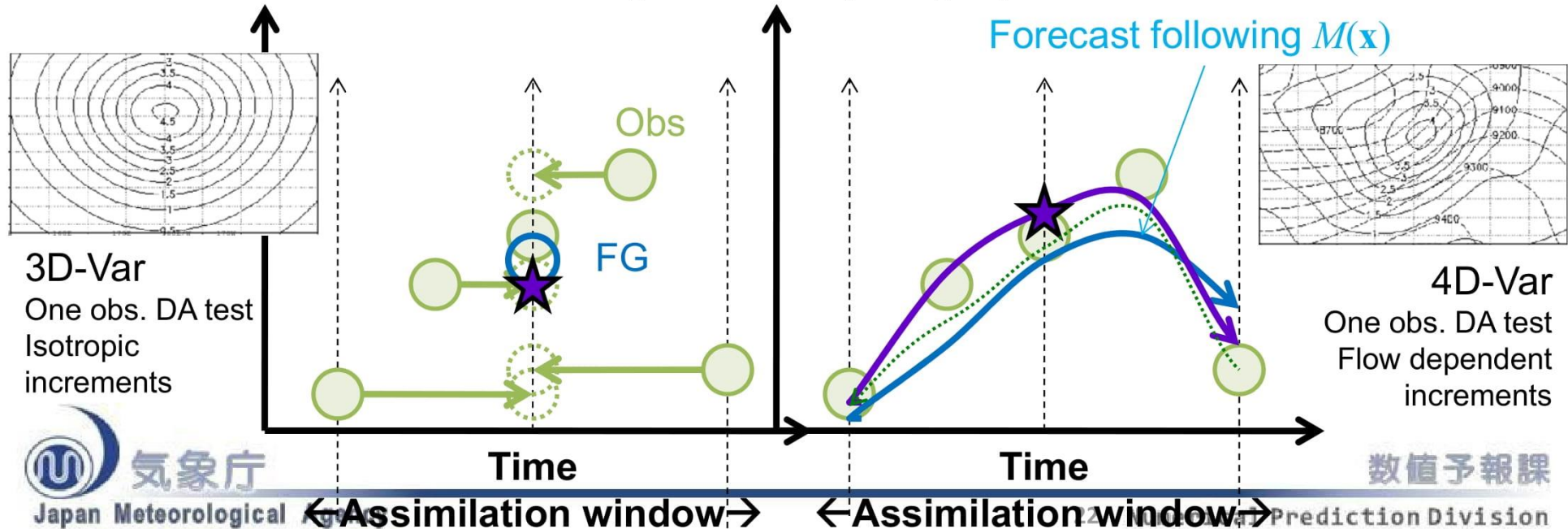
$$\delta WS = \frac{u\delta u + v\delta v}{\sqrt{u^2 + v^2}} = \begin{pmatrix} \frac{u}{\sqrt{u^2 + v^2}} & \frac{v}{\sqrt{u^2 + v^2}} \end{pmatrix} \begin{pmatrix} \delta u \\ \delta v \end{pmatrix}$$

Adjoint operator

$$\mathbf{H}^T = \begin{pmatrix} \frac{u}{\sqrt{u^2 + v^2}} \\ \frac{v}{\sqrt{u^2 + v^2}} \end{pmatrix} \begin{pmatrix} \overline{\delta u} \\ \overline{\delta v} \end{pmatrix} = \begin{pmatrix} \frac{u}{\sqrt{u^2 + v^2}} \\ \frac{v}{\sqrt{u^2 + v^2}} \end{pmatrix} \overline{\delta WS}$$

3D-Var/4D-Var

- 3D-Var: NOT consider the time evolution of the state
 - Obs. are compared with the GPV at the analysis time
 - It is supposed the obs. was performed at the analysis time
- 4D-Var: consider the time evolution of the state
 - Obs. are compared with the GPV, which are integrated to the observation time by model M . (FGAT: first guess at the appropriate time)
 - The difference is integrated back by using adjoint model M^T



Procedures of the variational method

– In case of Wind speed assimilation:

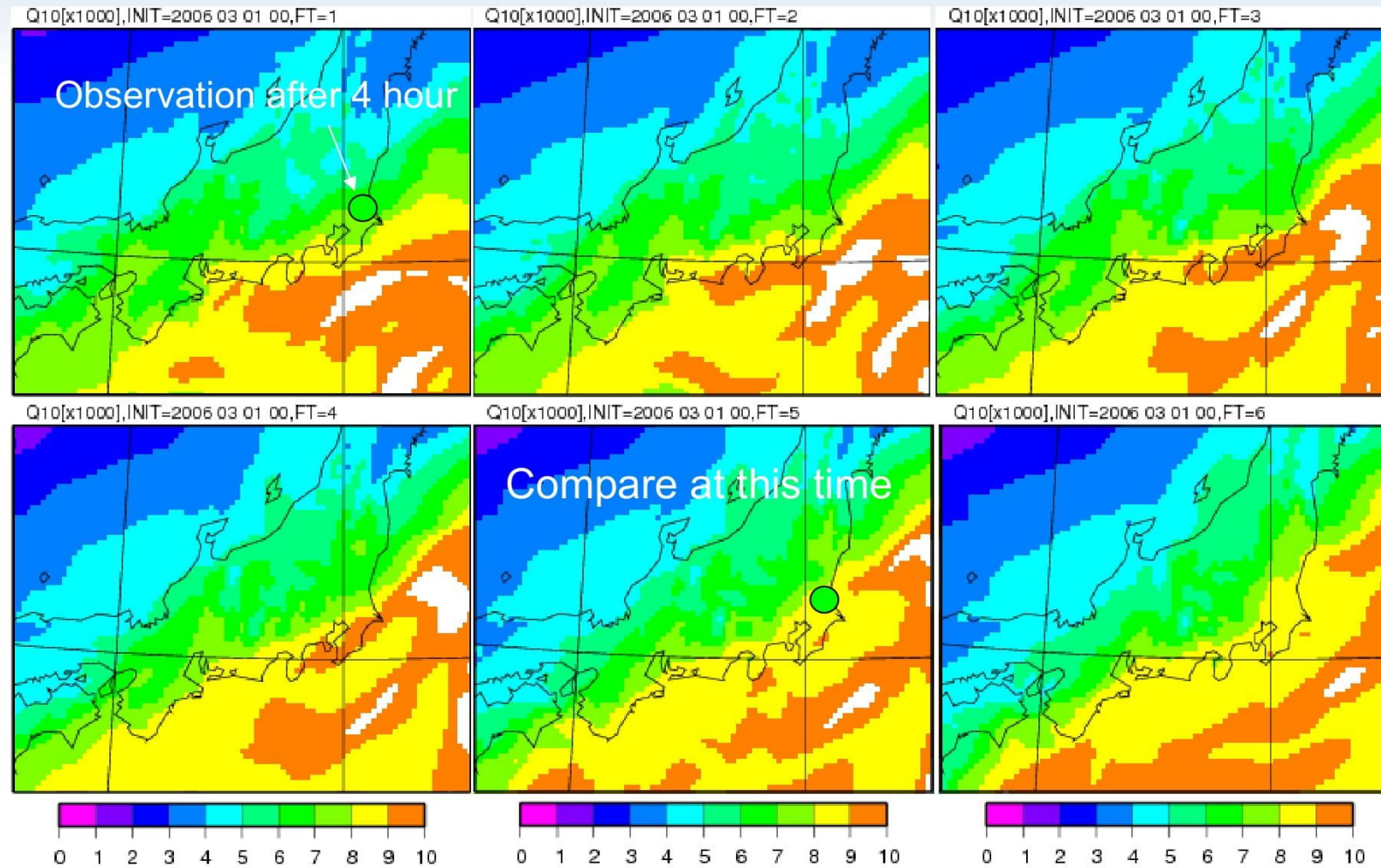
- Loop for cost function minimization
 - Convert analysis variables into forecast variables (ST, VP, ... \rightarrow U, V, ...)
 - # Integrate the GPVs to the observation time with the model
 - Interpolation of GPVs for the observation point
 - Calculate wind speed by using the interpolated u, v
 - Subtract the observation y, and divided by variance R
 - Estimate u' , v' by using adjoint operator for wind speed
 - Estimate GPVs' by using adjoint operator for interpolation
 - # Integrate back the GPVs' to the initial time with the adjoint model
 - Estimate analysis variables' by using adjoint operator for the conversion
 - Estimate the total gradient by adding the gradient from bkg term
 - » Take difference of \mathbf{x}_a and \mathbf{x}_b , multiplied by \mathbf{B}^{-1}
 - Use non linear optimization algorithm for estimating the correction amount.
 - » \rightarrow Exit if cost function J was converged. Otherwise, Repeat to the top.

is a step
for 4D-Var

- End after convert the variables.

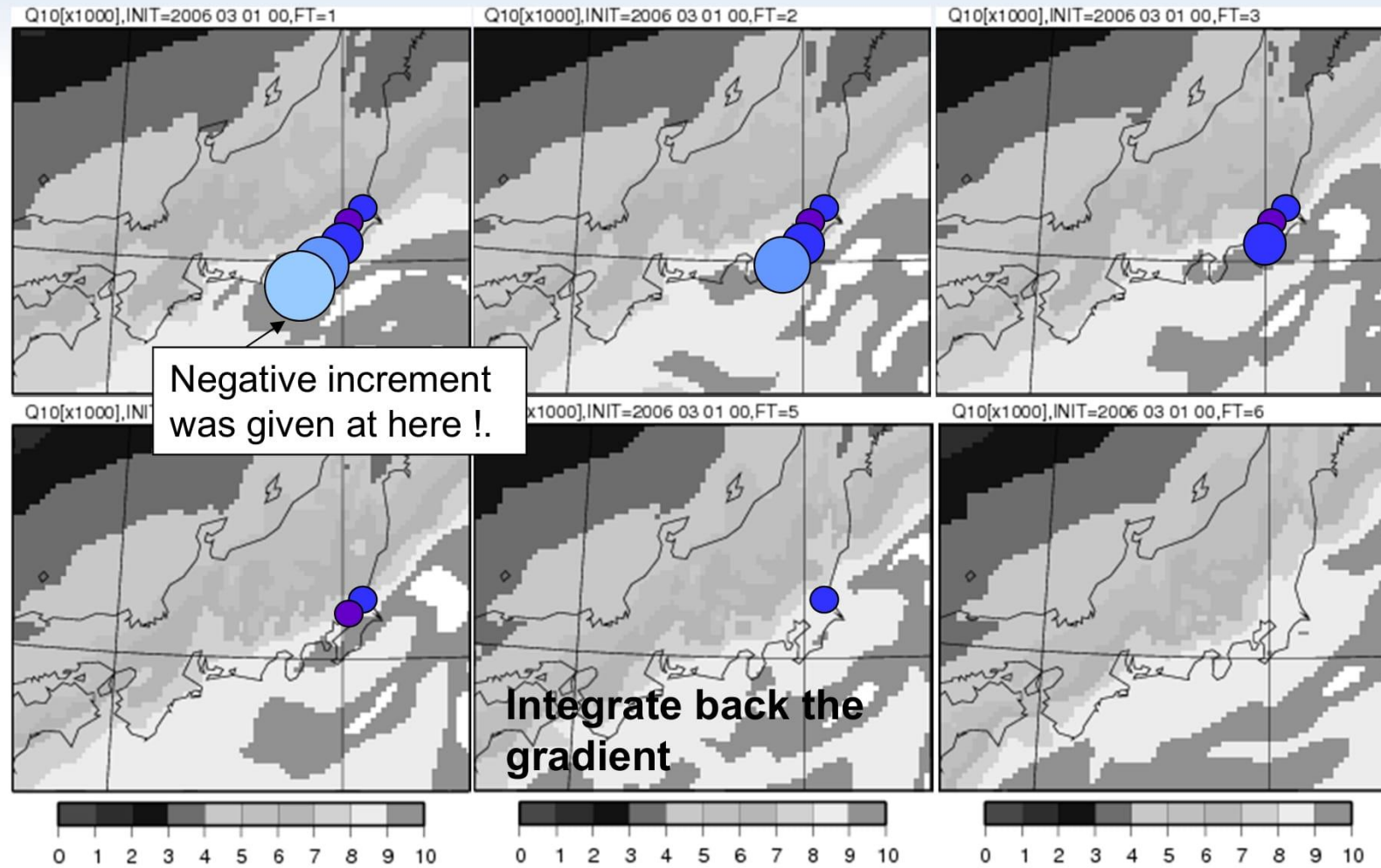
Schematic image of DA

for water vapor at 4 hours after



Schematic image of DA

for water vapor at 4 hours after



ENSEMBLE DA

– Ensemble Kalman Filter (EnKF) –

Kalman Filter

$$t = i - 1 \quad \mathbf{x}_{i-1}^a, \mathbf{P}_{i-1}^a$$

$$\begin{aligned} \text{Forecast} \quad \mathbf{x}_i^f &= M(\mathbf{x}_{i-1}^a) + \eta \\ \mathbf{P}_i^f &= \mathbf{M}\mathbf{P}_{i-1}^a\mathbf{M}^T + \mathbf{Q}_{i-1} \end{aligned}$$

$$t = i$$

$$\begin{aligned} \text{Analysis} \quad \mathbf{K}_i &= \mathbf{P}_i^f \mathbf{H}_i^T (\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R}_i)^{-1} \\ \mathbf{x}_i^a &= \mathbf{x}_i^f + \mathbf{K}_i (\mathbf{y}_i^o - H_i(\mathbf{x}_i^f)) \\ \mathbf{P}_i^a &= (\mathbf{I} - \mathbf{K}_i \mathbf{H}_i) \mathbf{P}_i^f \end{aligned}$$

Difficulties in KF

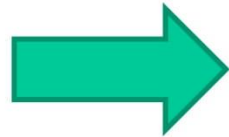
- **Dimension Problem**

- Calculation of inverse matrix

- ↑ Reduce the rank of observations (Serial filter)

- **Integration of Pf**

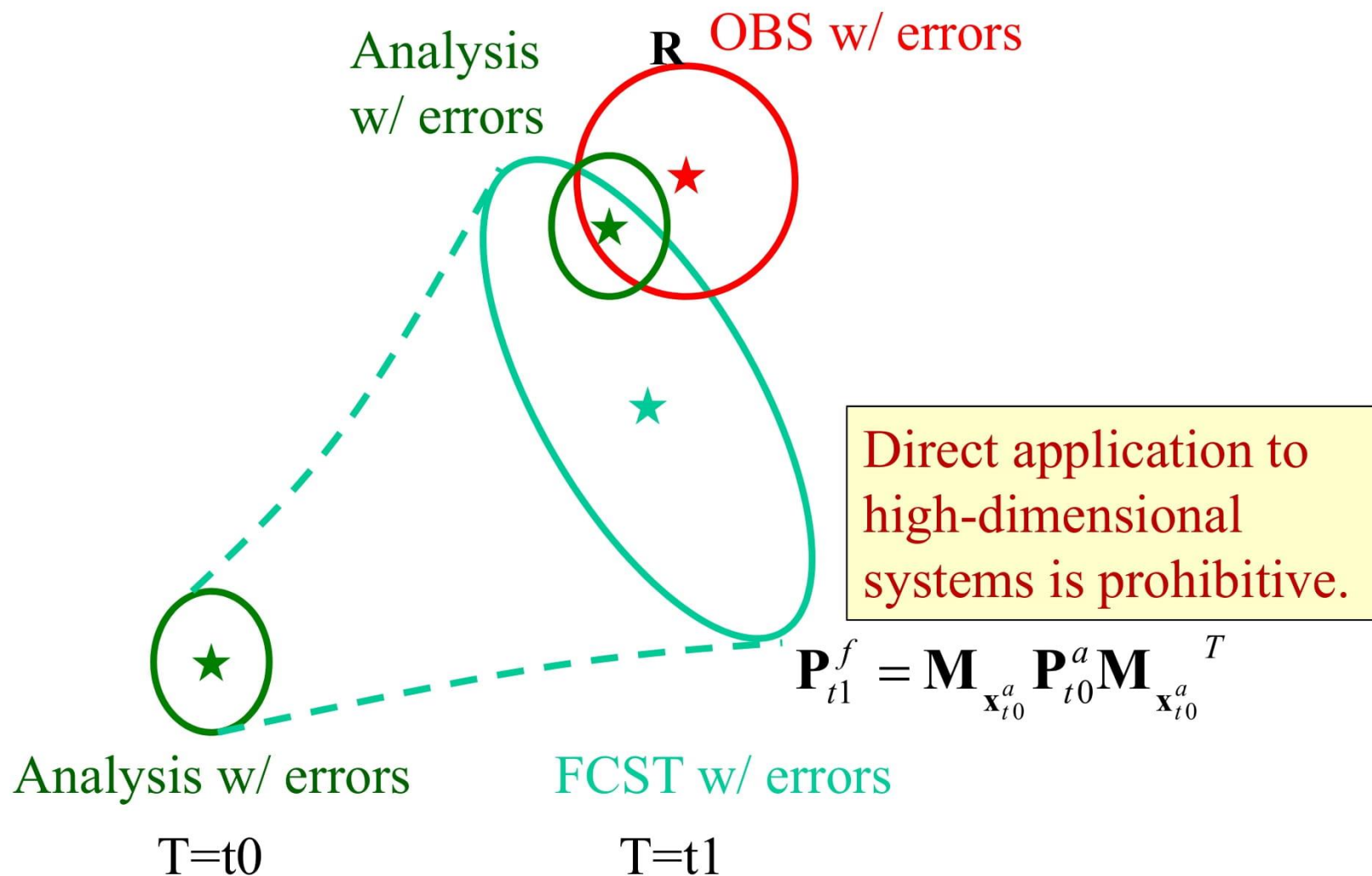
- Carry out the time integration by tangent linear model **2N times** ! ($N \doteq 10^7$)



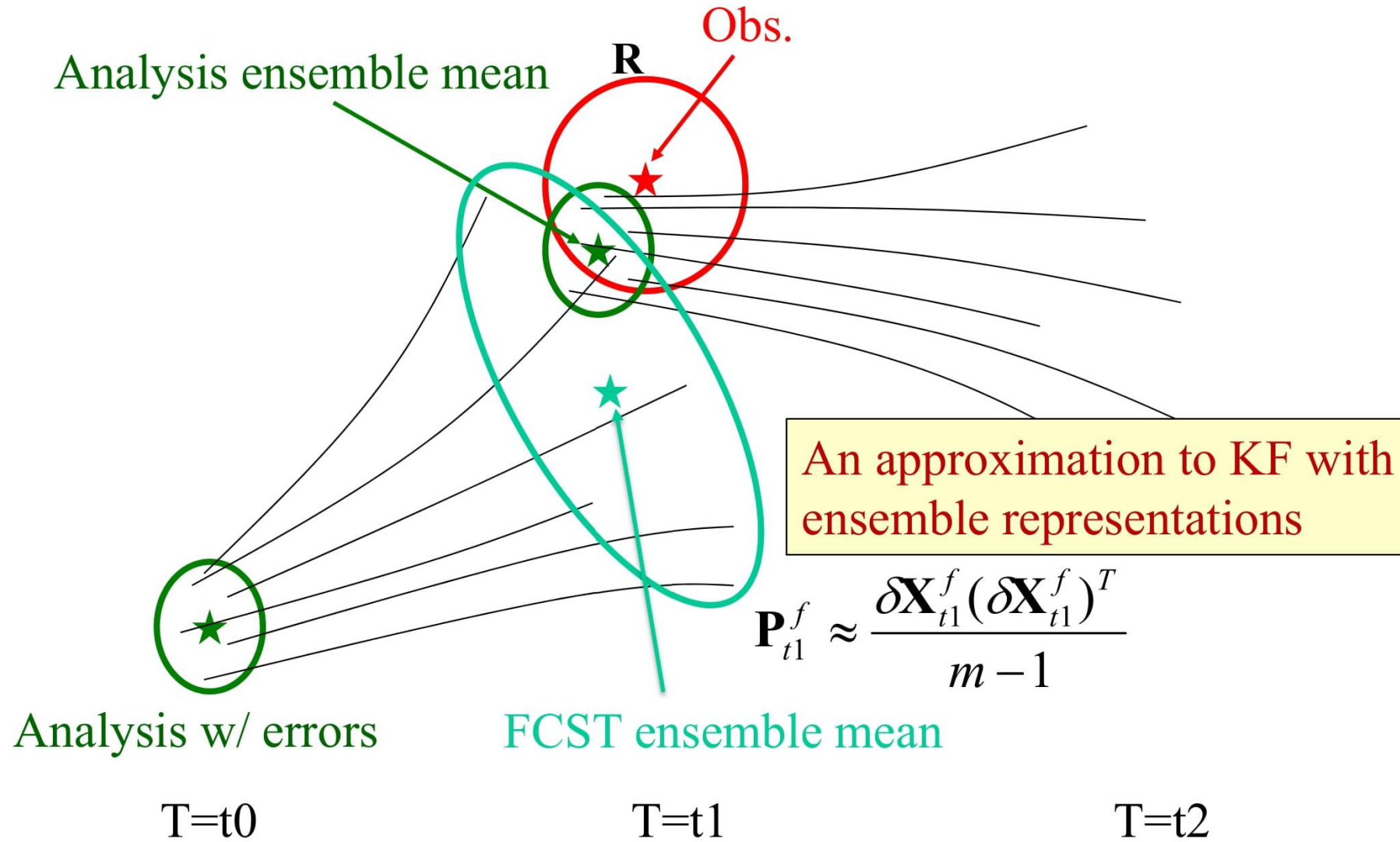
Reduce rank

SEEK filter, EnKF

Kalman Filter (KF)



Ensemble Kalman Filter (EnKF)



Approximation in KF

Approximation of the forecast error covariance matrix

$$\mathbf{P}_i^f = \left\langle \delta \mathbf{x}_i^f (\delta \mathbf{x}_i^f)^T \right\rangle \approx \frac{1}{m-1} \sum_{k=1}^m \left(\mathbf{x}_i^{f(k)} - \overline{\mathbf{x}}_i^f \right) \left(\mathbf{x}_i^{f(k)} - \overline{\mathbf{x}}_i^f \right)^T$$
$$\overline{\mathbf{x}}_i^f = \frac{1}{m} \sum_{k=1}^m \mathbf{x}_i^{f(k)}$$

Approximation of the Kalman gain matrix

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T (\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R}_i)^{-1}$$
$$\mathbf{P}_i^f \mathbf{H}_i^T = \left\langle \delta \mathbf{x}_i^f (\delta \mathbf{x}_i^f)^T \right\rangle \mathbf{H}_i^T \approx \frac{1}{m-1} \sum_{k=1}^m \left(\mathbf{x}_i^{f(k)} - \overline{\mathbf{x}}_i^f \right) \left(H_i \mathbf{x}_i^{f(k)} - \overline{H_i \mathbf{x}_i^f} \right)^T$$
$$\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T = \left\langle \mathbf{H}_i \delta \mathbf{x}_i^f (\mathbf{H}_i \delta \mathbf{x}_i^f)^T \right\rangle \approx \frac{1}{m-1} \sum_{k=1}^m \left(H_i \mathbf{x}_i^{f(k)} - \overline{H_i \mathbf{x}_i^f} \right) \left(H_i \mathbf{x}_i^{f(k)} - \overline{H_i \mathbf{x}_i^f} \right)^T$$
$$\overline{H_i \mathbf{x}_i^f} = \frac{1}{m} \sum_{k=1}^m H_i \mathbf{x}_i^{f(k)}$$

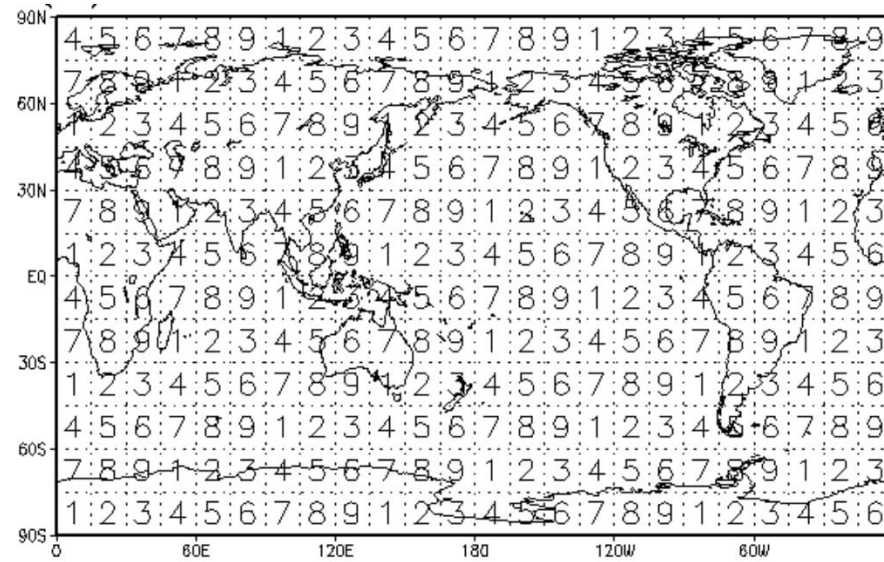
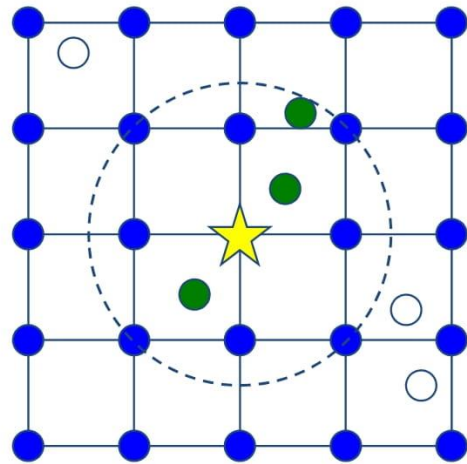
Analysis and ensemble update

- Perturbed Observation Method (PO)
 - Analysis for each member
 - Analysis error covariance are underestimated theoretically
- Square Root Filter (SRF)
 - Analysis for ensemble mean
 - Serial EnSRF, **LETKF**

LETKF (Local Ensemble Transform Kalman Filter)

Each grid point is treated **independently**.

→ *essentially perfectly parallel*



Multiple observations are treated **simultaneously**.

LETKF (Local Ensemble Transform Kalman Filter)

Kalman gain matrix:

$$\begin{aligned}\mathbf{K}_i &= \mathbf{P}_i^f \mathbf{H}_i^T (\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R}_i)^{-1} \\ &= \frac{1}{m-1} \mathbf{X}_i^f (\mathbf{X}_i^f)^T \mathbf{H}_i^T \left(\mathbf{H}_i \frac{1}{m-1} \mathbf{X}_i^f (\mathbf{X}_i^f)^T \mathbf{H}_i^T + \mathbf{R}_i \right)^{-1} \\ &= \mathbf{X}_i^f (\mathbf{Y}_i^f)^T \left[\mathbf{Y}_i^f (\mathbf{Y}_i^f)^T + (m-1)\mathbf{R}_i \right]^{-1} \\ &\quad \mathbf{y}_i^{f(k)} = H \left(\mathbf{x}_i^{f(k)} \right); \quad \mathbf{Y}_i^f = \left[\mathbf{y}_i^{f(1)} - \overline{\mathbf{y}_i^f} \mid \dots \mid \mathbf{y}_i^{f(m)} - \overline{\mathbf{y}_i^f} \right] = \mathbf{H} \mathbf{X}_i^f \\ &= \mathbf{X}_i^f \left[(m-1)\mathbf{I} + (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1} \mathbf{Y}_i^f \right]^{-1} (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1}\end{aligned}$$

LETKF (Local Ensemble Transform Kalman Filter)

Analysis error covariance in **ensemble space**:

$$\begin{aligned}\tilde{\mathbf{P}}_i^a &= \left[(m-1)\mathbf{I} + (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1} \mathbf{Y}_i^f \right]^{-1} & cf. \quad \mathbf{P}_i^a &= \left[(\mathbf{P}_i^f)^{-1} + \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{H}_i \right]^{-1} \\ \mathbf{K}_i &= \mathbf{X}_i^f \tilde{\mathbf{P}}_i^a (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1} = \mathbf{X}_i^f \tilde{\mathbf{K}}_i\end{aligned}$$

Ensemble mean update in **model space**:

$$\begin{aligned}\overline{\mathbf{x}}_i^a &= \overline{\mathbf{x}}_i^f + \mathbf{K}_i (\mathbf{y}_i^o - \overline{\mathbf{y}}_i^f) \\ &= \overline{\mathbf{x}}_i^f + \mathbf{X}_i^f \tilde{\mathbf{P}}_i^a (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1} (\mathbf{y}_i^o - \overline{\mathbf{y}}_i^f)\end{aligned}$$

LETKF (Local Ensemble Transform Kalman Filter)

Ensemble perturbations in **ensemble space**:

$$\mathbf{W}^a = [(m - 1) \tilde{\mathbf{P}}_i^a]^{1/2}$$

Ensemble perturbations in **model space**:

$$\begin{aligned} \mathbf{X}_i^a &= \mathbf{X}_i^f \mathbf{W}^a \\ &= \mathbf{X}_i^f [(m - 1) \tilde{\mathbf{P}}_i^a]^{1/2} \end{aligned}$$

LETKF (Local Ensemble Transform Kalman Filter)

In practice, the implementation of the LETKF algorithm requires the following steps:

$$\mathbf{Y}_i^f = \mathbf{H}\mathbf{X}_i^f$$

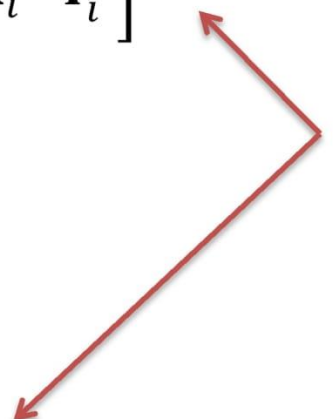
$$\tilde{\mathbf{P}}_i^a = \left[(m-1)\mathbf{I} + (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1} \mathbf{Y}_i^f \right]^{-1}$$

$$\mathbf{K}_i = \mathbf{X}_i^f \tilde{\mathbf{P}}_i^a (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1}$$

$$\overline{\mathbf{x}}_i^a = \overline{\mathbf{x}}_i^f + \mathbf{K}_i (\mathbf{y}_i^o - \overline{\mathbf{y}}_i^f)$$

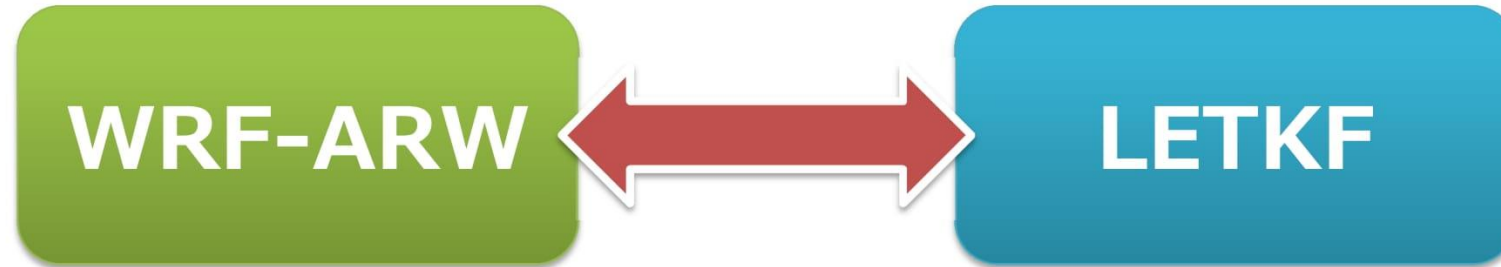
$$\mathbf{X}_i^a = \mathbf{X}_i^f \left[(m-1)\tilde{\mathbf{P}}_i^a \right]^{1/2}$$

$$\mathbf{x}_i^{a(k)} = \overline{\mathbf{x}}_i^a + \mathbf{X}_i^a \mathbf{x}_i^{a(k)}$$

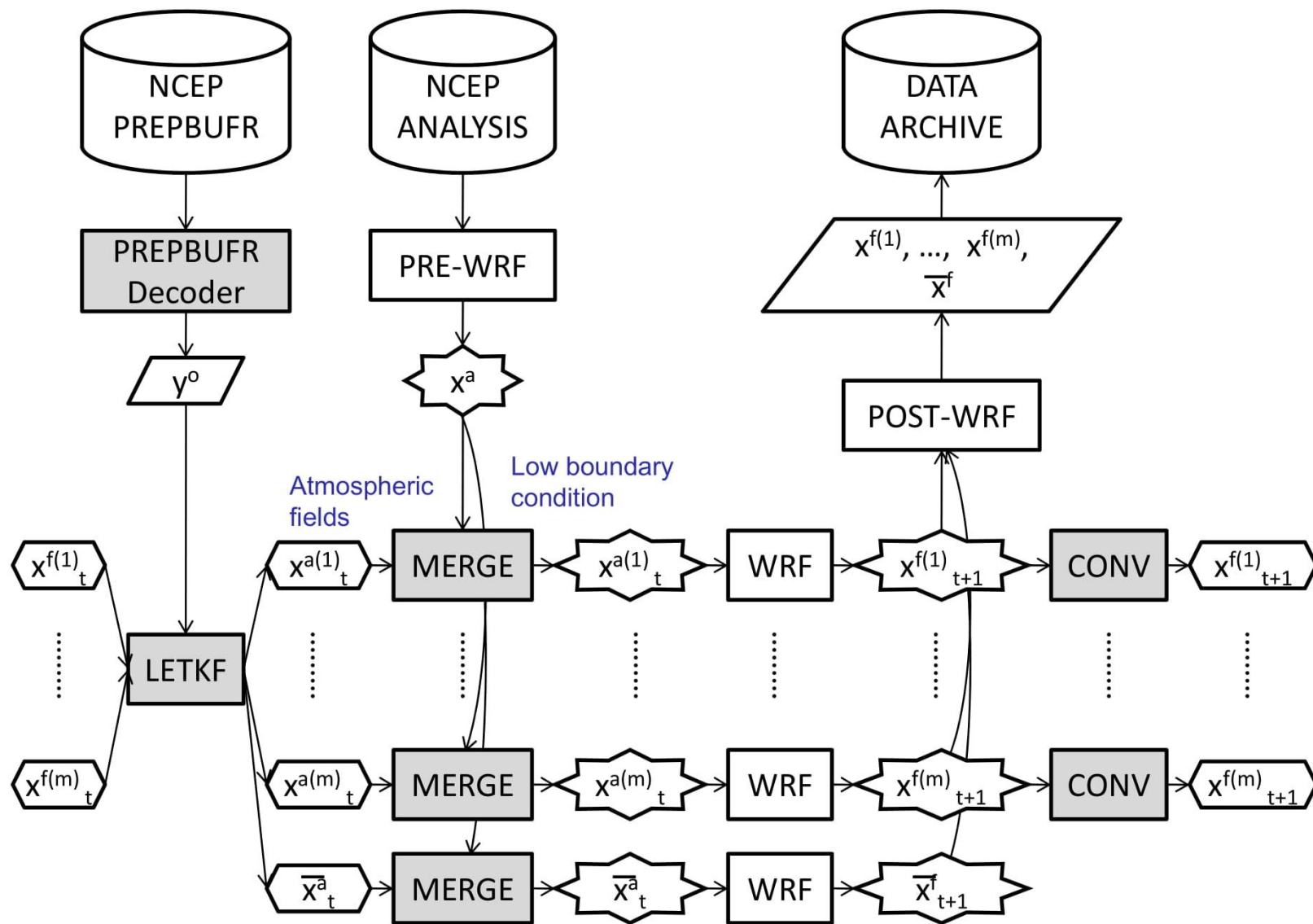
$$(m-1)\mathbf{I} + (\mathbf{Y}_i^f)^T \mathbf{R}_i^{-1} \mathbf{Y}_i^f = \mathbf{U}\mathbf{D}\mathbf{U}^T$$


WRF-LETKF

Community Model



- **WRF-ARW**
 - See WRF-ARW TUTORIALS
<http://www.mmm.ucar.edu/wrf/users/supports/tutorial.html>
- **LETKF**
 - Available at:
<http://code.google.com/p/miyoshi/>



Experimental settings

- *LETKF settings*

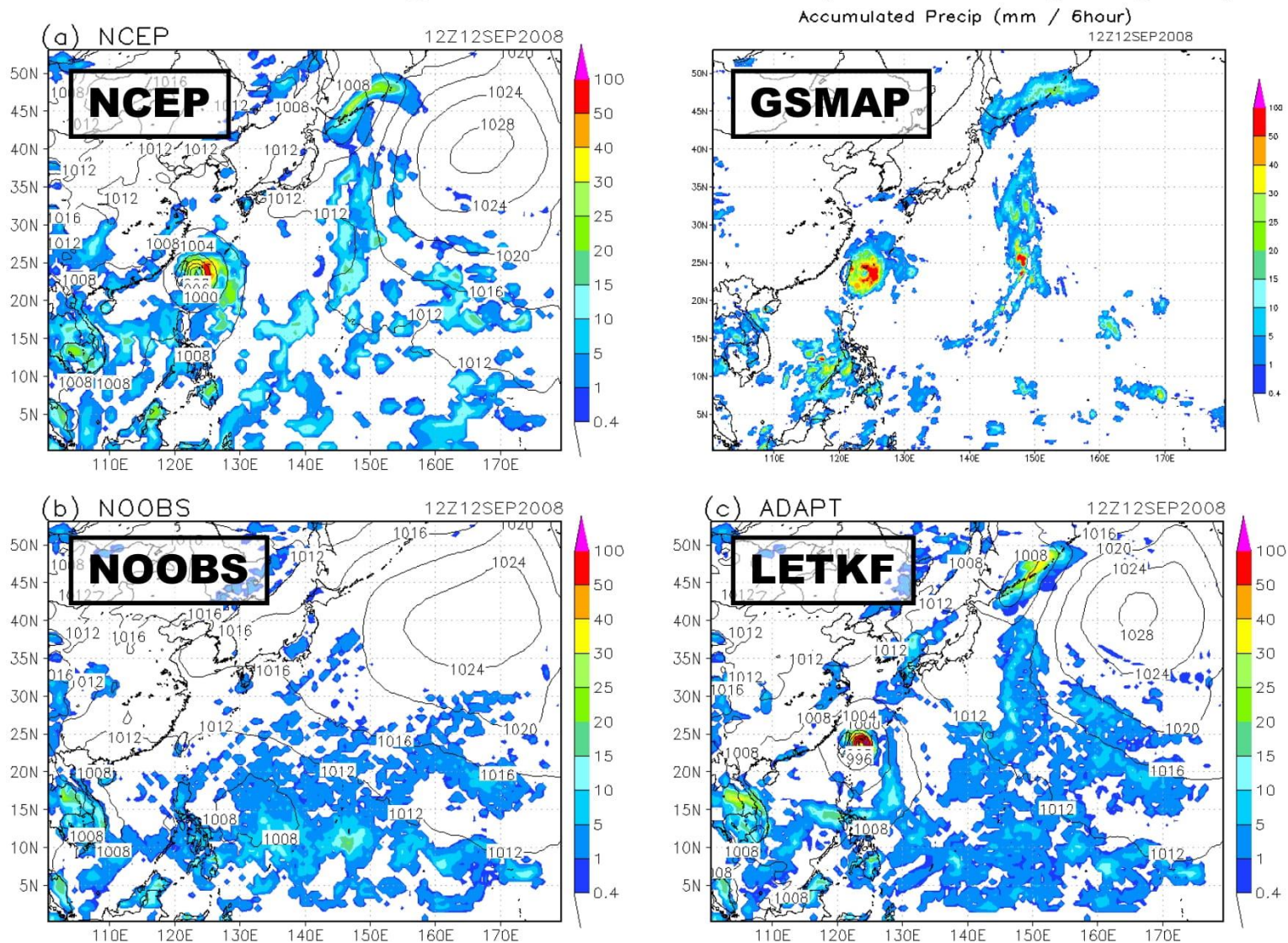
Ensemble size	20
Lateral boundary conditions	Unperturbed
Covariance inflation	Adaptive (Miyoshi 2010) Fixed 20% (smaller above level 20)
Covariance localization	400 km, 0.4 ln p
Analyzed variables	u, v, w, T, ph, qv, qc, qr
Observation data	NCEP PREPBUFR

- *WRF settings*

Domain size	137 x 109 x 40
Horizontal grid spacing	~ 60 km
WRF version	WRF-ARW 3.2.1

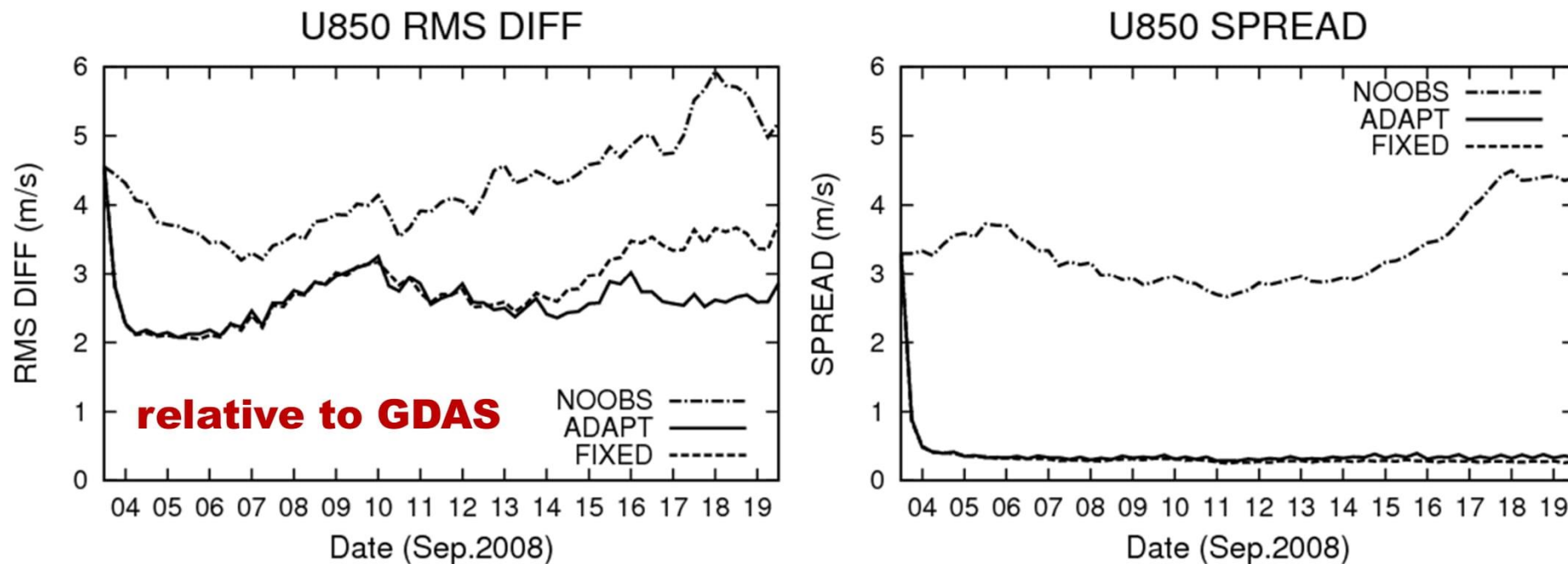
WRF-LETKF working properly

WRF 6-h forecast using each initial field (after 9 days cycle)



NOOBS: without observations (BC is updated every 6-h)

Time series (U850)



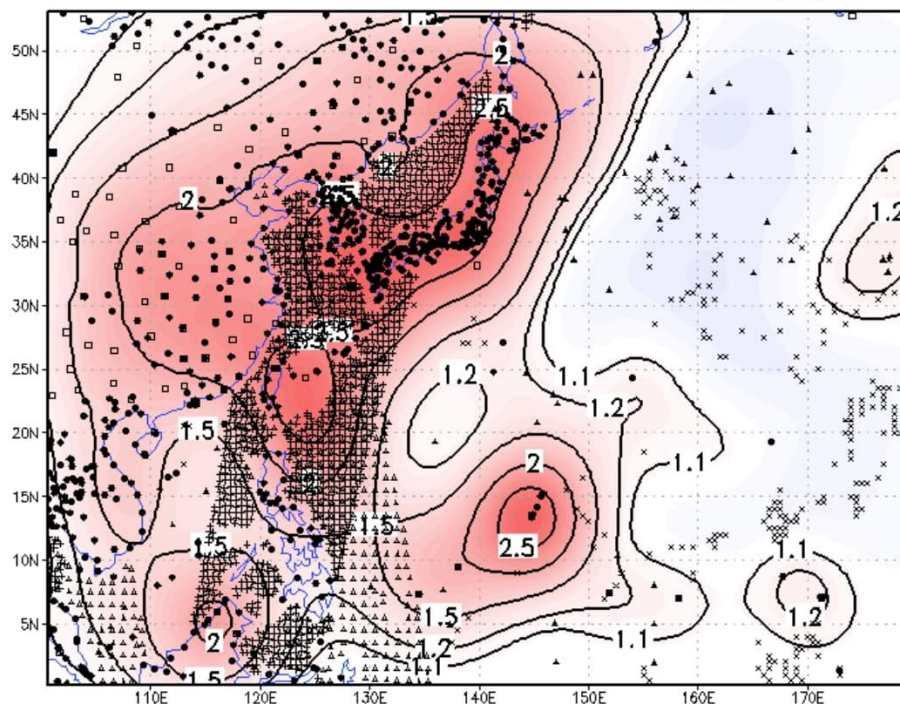
- LETKF performs properly.
- ADAPT shows significantly smaller RMS after 10 days cycle.
 - It takes a while to spin-up the adaptive inflation parameters
- **Lateral boundary** does not dominate the inner domain.

Adaptive inflation

Lower troposphere

Multiplicative Inflation Factor (lev = 8)

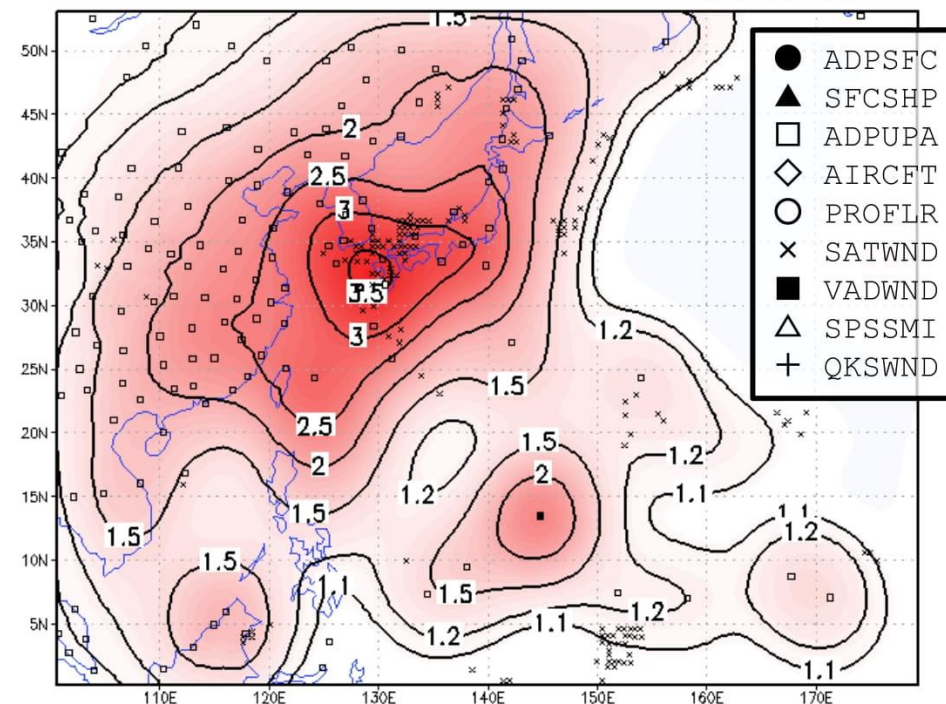
12Z12SEP2008



Middle troposphere

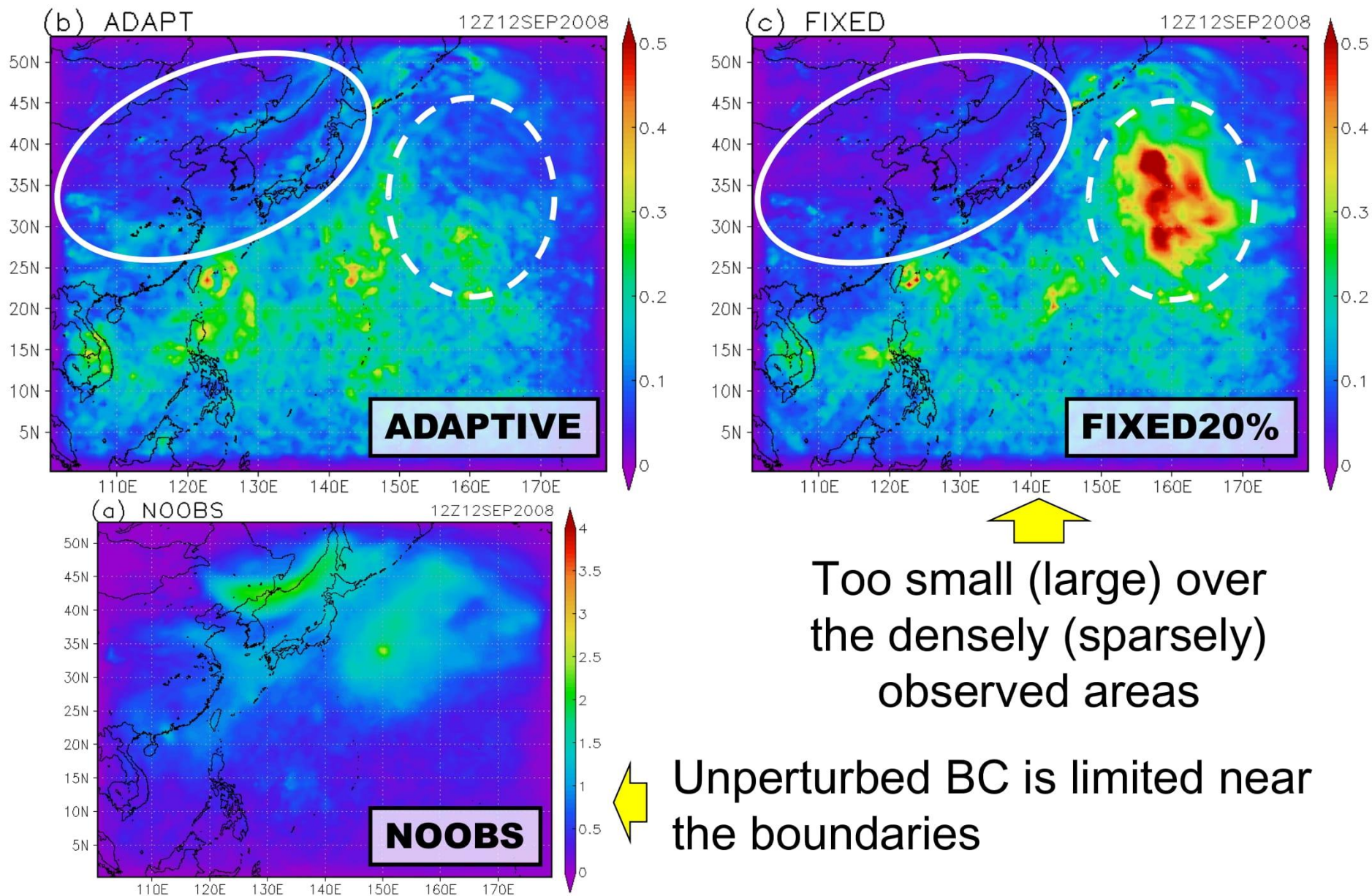
Multiplicative Inflation Factor (lev = 15)

12Z12SEP2008

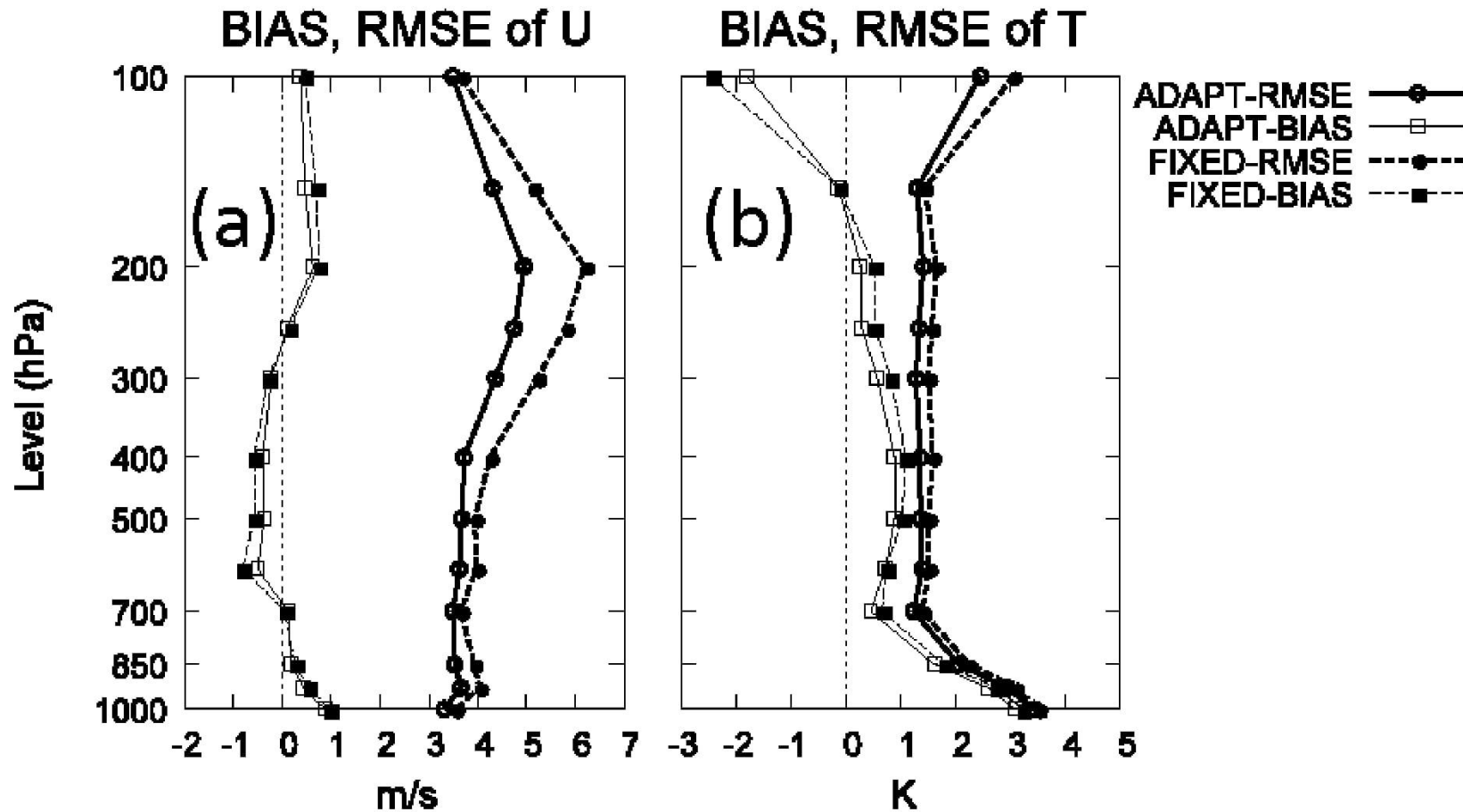


- Adaptive inflation accounts for imperfections such as **model errors** and **limited ensemble size**.
- The large adaptive inflation values are estimated over the densely observed areas, which make the ensemble spread significantly larger in the densely observed areas.

Ensemble spread (T500)

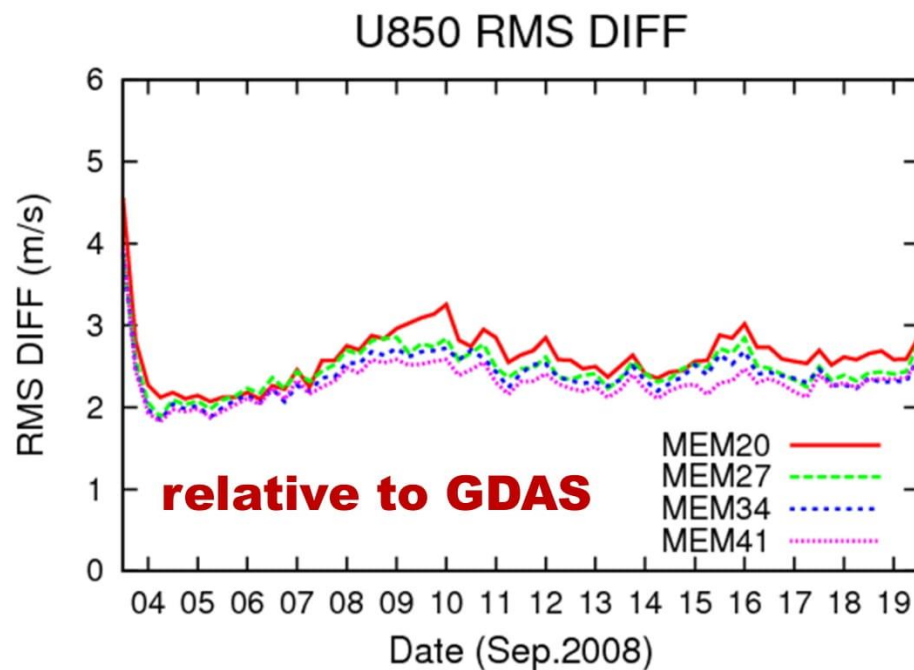


6-hr forecast vs. radiosondes

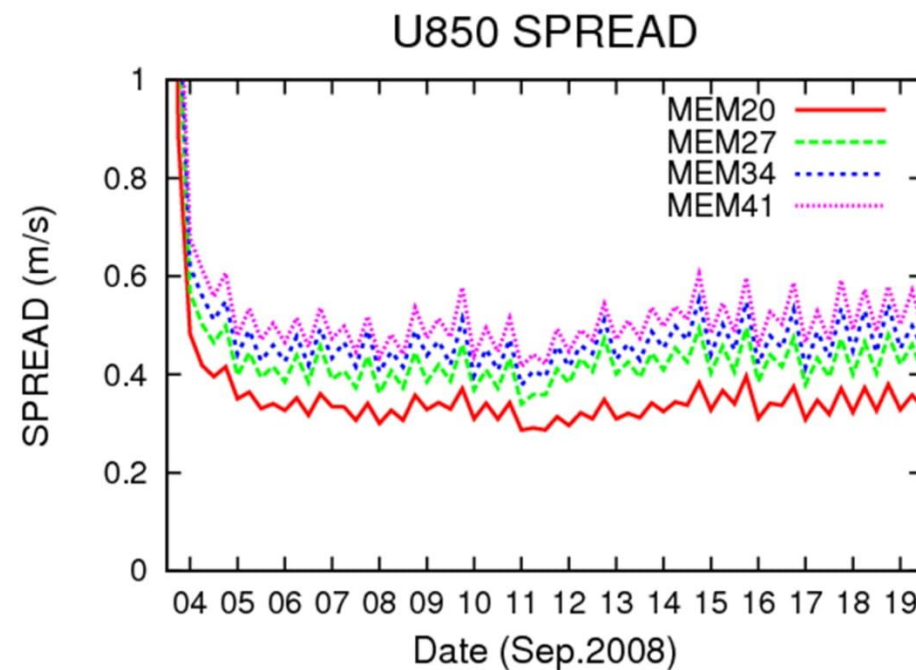


➤ Adaptive inflation performs well.

Sensitivity to the ensemble size

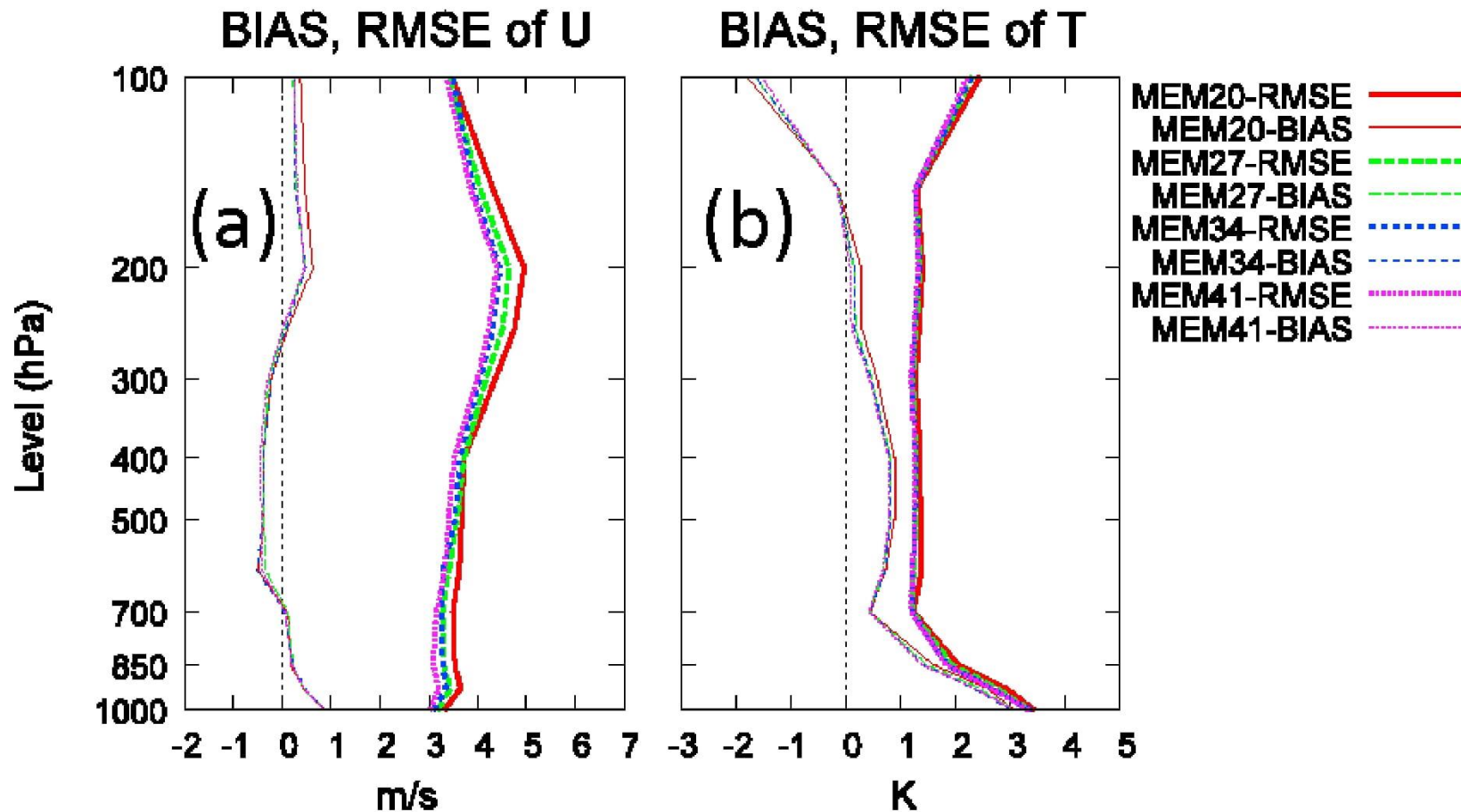


➤ Consistently better with more members.



➤ More members, the larger the ensemble spread.

Sensitivity to the ensemble size



- Consistently better with more members.
- For temperature, increasing the ensemble size more than 27 does not show much improvement.

Conclusions

- The WRF-LETKF system performed properly with real observations.
- **Adaptive inflation** performed very well.
- More ensemble members yielded consistently better analyses.
 - Roughly linear increase of the computational time